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# Sample Complexity of Networked Control Systems over Unknown Channels

Konstantinos Gatsis and George J. Pappas

*Abstract*— **Recent control trends are increasingly relying on communication networks and wireless channels to close the loop in Internet-of-Things applications. Traditionally these approaches are modeled-based, i.e., given a network or channel model they analyze stability and design appropriate controller structures. However such modeling is a fundamental challenge as channels are typically unknown a priori and only available through data samples. In this work we aim to characterize the amount of channel modeling that is required to determine the stability of networked control tasks. Our most significant finding is a direct relation between the sample complexity and the system stability margin, i.e., the underlying packet success rate of the channel and the spectral radius of the dynamics of the control system.**

## I. Introduction

Wireless communication is increasingly used in autonomous applications to connect devices in industrial control environments, teams of robotic vehicles, and the Internet-of-Things. To guarantee safety and control performance it is customary to include a model of the wireless channel, for example an i.i.d. or Markov link quality, alongside the model of the physical system to be controlled. In such modeled-based approaches one can characterize, for example, that it is impossible to estimate or stabilize an unstable plant if its growth rate is larger than the rate at which the link drops packets [1]–[3], or below a certain channel capacity [4], [5]. Models also facilitate the allocation of communication resources to optimize control performance in, e.g., power allocation and scheduling over fading channels [6]–[8], or in event-triggered control [9]–[11].

In practice wireless autonomous systems operate under unpredictable channel conditions following unknown distributions, which are more often observable via a finite amount of collected channel sample measurements [12], [13]. The purpose of this work is the analysis of networked control systems when only channel sample data are available instead of channel models. We use the data to learn whether a given networked control system is stable, and we also characterize how the learning procedure depends on the amount of channel samples and the control system parameters. To the best of our knowledge, our paper is the first to consider data-based algorithms and sample complexity analysis for networked control, in contrast to the vast literature on model-based approaches mentioned above.

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Learning methods have been used in control problems most commonly within the reinforcement learning and approximate dynamic programming literature [14], [15], where the goal is to learn from data how to control when the dynamics of the system are unknown. One approach within this framework is based on learning the system dynamics [16]. This is used for example in analyzing the sample complexity of the classic multi-armed bandit problem [17], [18] and more recently in the quadratic control of unknown linear systems [19]–[22]. Very recently deep reinforcement learning has been examined for allocation of communication resources in control systems [23], [24]. In contrast, to the best of our knowledge our work is the first one focused on learning the channel model instead of system dynamics and considers explicitly the sample complexity of networked control systems. We also point out that an alternative approach is to bypass building channel models altogether and learn solutions directly as in our previous work on power allocation in [25], [26] and multiple-access in [8], [27]. Sampling methods were also very recently used to analyze the stability of a switched linear system under arbitrary switching [28], which is a related but different problem than the stability analysis considered here under fixed but unknown switching induced by the channel.

We consider the stability of a linear dynamical system over a Bernoulli packet-dropping channel with an unknown success rate (Section II). Using channel sample data, i.e., a number of packet successes and failures, we develop an algorithm to learn whether the networked control system is stable or not (Section III). To do this we utilize confidence bounds obtained by concentration inequalities, more specifically, Hoeffding's inequality. As our algorithm depends on random channel samples there is always a probability of error, i.e., the algorithm determines that the system is stable while the true system is not. We characterize the statistical properties of the algorithm (Theorem 1) as well as the amount of channel sample data needed to correctly learn the system stability and control performance with a desired confidence level.

Our most significant finding is that the sample complexity adversely depends on the system stability margin, i.e., the underlying packet success rate of the channel and the spectral radius of the system. A significantly larger number of samples is needed if the networked control system over the channel is closer to instability. This means that it becomes impractical to verify stability under a large range of plant and channel configurations. The derived sample complexity can be practically useful in describing the amount of channel samples required if we are willing to verify stability with

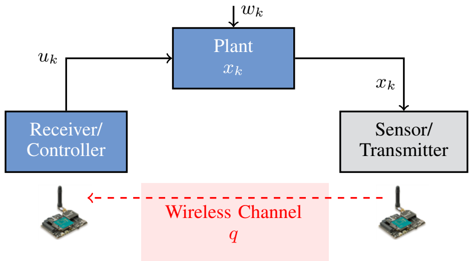


Fig. 1. Wireless Control System. A sensor measures the state of a plant perturbed by a random disturbance. The sensor transmits the measured information over a packet-dropping wireless channel to a receiver/controller providing control inputs.

high confidence up to a certain system stability margin. We validate our theoretical analysis in numerical simulations (Section IV).

## II. Modeled-based Networked Control

We consider the evolution of a dynamical system over a packet dropping channel. This is a standard model for remote estimation or control over a network or a wireless channel, for example when a sensor measures the state of the plant and transmits it to a receiver – see Fig. 1 and [1]–[3] for related examples. Our goal is to analyze the stability properties and the control performance of the system, hence we assume the dynamics for the system are fixed, for example a controller has been already designed. We assume that the evolution of the system depends on whether a transmission occurs at time $k$ or not, indicated with variables $\gamma_k \in \{0, 1\}$. We suppose the system evolution is described by a switched linear time invariant model of the form

$$x_{k+1} = \begin{cases} A\,x_k + w_k, & \text{if } \gamma_k = 0 \\ w_k, & \text{if } \gamma_k = 1 \end{cases} . \tag{1}$$

Here $x_k \in \mathbb{R}^n$ denotes the state of the overall control system at each time $k$. At a successful transmission the system dynamics are reset to zero, and otherwise when the transmission fails the dynamics are in open loop described by $A \in \mathbb{R}^{n \times n}$. The open loop matrix $A$ may be unstable, i.e., the eigenvalue with the largest modulus may be larger than unity, $\rho(A) = \max_{i=1,\dots,n} |\lambda_i(A)| > 1$. This case motivates the stability analysis. The additive terms $w_k, k \geq 0$ model an independent identically distributed (i.i.d.) noise process across time with mean zero and positive definite covariance $W$.

We will employ the usual quadratic system state cost at each time step $k$ as $x_k^T Q\, x_k$, where $Q$ is a positive definite matrix. Intuitively there is a higher penalty when the state of the system is away from the origin. The cost over time depends on whether the transmissions are successful or not over time. In this paper we make the assumption that $\{\gamma_k, k \geq 0\}$ are independent Bernoulli random variables with a constant success probability $q$, and they are also independent from the system state $x_k$ and noise $w_k$. This i.i.d. assumption is very crucial for our results as we discuss in Remark 1.

Given the model of the transmission success we can describe the effect on the control system performance with the average quadratic cost

$$J(q) = \limsup_{N \to \infty} \frac{1}{N} \sum_{k=0}^{N-1} \mathbb{E}[x_k^T\, Q\, x_k]. \tag{2}$$

The expectation at the right hand side accounts for the randomness introduced by the system disturbance and the channel. We choose to denote this cost as a function of the success rate of the channel $q$ but it also depends on the control system parameters $A, W$.

When the channel success rate $q$ is known, we have the following result.

**Lemma 1.** *Consider the switched linear system (1) over an i.i.d. Bernoulli binary channel with a known success probability $q \in [0, 1]$. Then:*

1) *The system is stable ($\sup_k \mathbb{E} x_k x_k^T < \infty$) if and only if*
$$q > 1 - \frac{1}{\rho(A)^2} \tag{3}$$
2) *If condition (3) holds, the quadratic control cost (2) is a non-increasing function of the success rate $q$ given by*
$$J(q) = Tr(PW) \tag{4}$$
*where $P$ is the unique positive definite solution of the (Lyapunov) matrix equation*
$$P = Q + (1 - q) A^T P A. \tag{5}$$

*Proof.* The stability condition (3) as well as the expression for the control cost in (4)-(5) follow from the random jump linear system theory [29].

The fact that the control cost (2) is a decreasing function can be shown as follows. It is straightforward to show that the solution to the Lyapunov equation (5) can be written as

$$P = \sum_{i=0}^{\infty} (1 - q)^i\, (A^i)^T Q A^i \tag{6}$$

where the sum converges due to (3). Plugging in this expression in (4) yields the expression for the control cost as

$$J(q) = \sum_{i=0}^{\infty} (1 - q)^i\, \mathrm{Tr}((A^i)^T Q A^i W) \tag{7}$$

Taking the derivative with respect to $q$ verifies that the function is non-increasing because all terms $\mathrm{Tr}((A^i)^T Q A^i W)$ are non-negative. □

The above result is a fundamental limit in the sense that it characterizes the absolute minimum channel success rate required for stability as a function of the eigenvalues of the dynamics $A$. The Lemma also gives an expression for the control performance as a function of the system and channel parameters.

However in practice the channel success rate $q$ is unknown. Instead we may have access to channel sample data. The problem we would like to answer is twofold:

- how to check whether the system is stable or not using the channel sample data?
- what is the confidence of the mehod and how does it scale with the amount of channel samples and the control system and channel parameters?

## III. Sample-based Stability Analysis of Networked Control

Suppose that instead of knowing the packet success rate $q$ of the channel we have available $N$ samples denoted by $\gamma_k, k = 0, \ldots, N-1$ drawn independently from the Bernoulli distribution with success $q$. In practice this data is easy to obtain, it suffices to send $N$ packets and record whether they are received or not. Given this data the most natural approximation of the true success probability $q$ is the sample average

$$\hat{q}_N = \frac{1}{N}\sum_{k=0}^{N-1}\gamma_k \tag{8}$$

Indeed this approximation is in some sense optimal - the sample average is the maximum likelihood estimate given the data.

In the case of unlimited data samples the sample average converges almost surely to the true underlying packet success rate by the Strong Law of Large Numbers [30, Ch.2]. Hence with unlimited data, learning the stability of the control system, i.e., checking whether (3) holds, would be easy. In practice only finite amount of data will be available and this motivates us to investigate a finite sample analysis.

For a finite number of samples we argue that instead of point estimates of the channel success rate, confidence intervals are more useful. One easy approach to construct confidence intervals by the channel sample data is using concentration inequalities. In particular in this paper we employ Hoeffding's inequality.

**Lemma 2.** *[Hoeffding's inequality, Th. 2.8 [31]] Consider a sequence* $\{\gamma_k, k = 0, \ldots, N-1\}$ *of i.i.d. random variables taking values in* $[0, 1]$ *with mean q. Let* $\hat{q}_N = \frac{1}{N}\sum_{k=0}^{N-1}\gamma_k$ *be the sample average. Then for any* $\varepsilon > 0$ *we have that*

$$\mathbb{P}(\hat{q}_N \geq q + \varepsilon) \leq \exp\{-2N\varepsilon^2\} \tag{9}$$

*where the probability is with respect to the random sequence* $\{\gamma_k, k = 0, \ldots, N-1\}$.

The result essentially states that there is a low probability that the sample average deviates much from the true packet success rate and further provides an explicit bound on this probability.

There is a useful direct consequence of this inequality. Given a desired high confidence bound $1 - \delta$ where $\delta$ is a small positive value, for example of the order of $10^{-3}$, and after collecting $N$ samples, we may derive a confidence about the true underlying mean, that is, the channel success rate in our problem, as follows.

**Lemma 3.** *Consider a sequence* $\{\gamma_k, k = 0, \ldots, N-1\}$ *of i.i.d. random variables taking values in* $[0, 1]$ *with mean q. Let* $\hat{q}_N = \frac{1}{N}\sum_{k=0}^{N-1}\gamma_k$ *be the sample average. Then for any* $\delta \in (0, 1)$ *it holds that*

$$\mathbb{P}\left(q \geq \hat{q}_N - \sqrt{\frac{\log(1/\delta)}{2N}}\right) \geq 1 - \delta \tag{10}$$

*where the probability is with respect to the random sequence* $\{\gamma_k, k = 0, \ldots, N-1\}$.

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**Algorithm 1** Stability and performance analysis using channel samples

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**Input:** Dynamics $A$, Noise covariance $W$, Confidence level $\delta$, Number of samples $N$, Channel samples $\gamma_0, \ldots, \gamma_{N-1} \in \{0, 1\}^N$

Compute the sample average

$$\hat{q}_N = \frac{1}{N}\sum_{k=0}^{N-1}\gamma_k \tag{11}$$

Compute the high confidence lower and upper bounds

$$q_{\min} = \hat{q}_N - \sqrt{\frac{\log(1/\delta)}{2N}} \tag{12}$$

$$q_{\max} = \hat{q}_N + \sqrt{\frac{\log(1/\delta)}{2N}} \tag{13}$$

**if** $1 - \frac{1}{\rho(A)^2} < q_{\min}$ **then**

  Solve for the matrix $P$ satisfying

$$P = Q + (1 - q_{\min})A^T P A. \tag{14}$$

  Compute $\hat{J}_N = \mathrm{Tr}(PW)$

  **return** 'Stable' and $\hat{J}_N$

**else**

  **if** $1 - \frac{1}{\rho(A)^2} > q_{\max}$ **then**

    **return** 'Unstable'

  **else**

    **return** 'Undetermined'

  **end if**

**end if**

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In this lemma the quantity $\hat{q}_N - \sqrt{\frac{\log(1/\delta)}{2N}}$ is a sample-based high-confidence lower bound on the true packet success rate of the channel.

Note that the result holds regardless of the distribution as long as it has a bounded support on $[0, 1]$. In particular in the case that we consider the result can be strengthened as the distribution of the sum of i.i.d. Bernoulli random variables $\sum_k \gamma_k$ is binomial so tighter confidence intervals can be computed [32]. Here we opt for the bound above for simplicity. In numerical simulations we will also examine its conservativeness.

### A. Stability Analysis Using Channel Samples

Let us now return to the main question of this paper. Given some channel data we would like to verify whether the system is stable, that is, whether the inequality (3) holds. We propose to utilize Hoeffding's inequality. We can construct an interval where the channel success rate lies with a desired high confidence using Lemma 3. Then we can check whether stability holds for all such high-confidence channel conditions. In particular it suffices to check whether stability holds for the lower end of this interval. A symmetric argument can verify instability of the system. This procedure is summarized in Algorithm 1.

The algorithm may not be able to determine stability or instability for every instance of the data. Intuitively this will occur if the sample mean of the data is sufficiently different from the true mean. Our main result is to analyze the average performance of Algorithm 1 using a finite number of samples. By average performance, we mean how often would the algorithm return the correct answer if we were to run it multiple times over independent data samples.

Let $A_N$ denote the event that algorithm provides the correct answer. That is, if the system is stable, i.e., condition $q > 1 - \frac{1}{\rho(A)^2}$ holds, then define $A_N$ as the event that Alg. 1 returns 'Stable'. Alternatively if the system is strictly unstable, i.e., condition $q < 1 - \frac{1}{\rho(A)^2}$ holds, then define $A_N$ as the event that Alg. 1 returns 'Unstable'. Similarly define as $B_N$ the event that the algorithm returns the incorrect answer, 'Unstable' in the first case, or 'Stable' in the latter. Otherwise it returns 'Undetermined'.

**Theorem 1.** *Consider the switched linear system (1) over an i.i.d. Bernoulli binary channel with an unknown success probability $q \in [0,1]$ and assume $q \neq 1 - \frac{1}{\rho(A)^2}$. Consider the stability analysis procedure developed in Algorithm 1 using $N$ i.i.d. channel samples drawn with success rate $q$. Let $A_N$ denote the event that the algorithm provides the correct answer . Then for all $N = 1, 2, \ldots$*

$$\mathbb{P}(A_N) \geq 1 - \exp\left\{-2N\left[\left|q - 1 + \frac{1}{\rho(A)^2}\right| - \sqrt{\frac{\log(1/\delta)}{2N}}\right]_+^2\right\} \tag{15}$$

*where $[\ ]_+$ denotes the projection to the positives, and the probability is with respect to the random channel samples. Moreover let $B_N$ denote the event that the algorithm provides the incorrect answer. Then for all $N = 1, 2, \ldots$*

$$\mathbb{P}(B_N) \leq \exp\left\{-2N\left[\left|q - 1 + \frac{1}{\rho(A)^2}\right| + \sqrt{\frac{\log(1/\delta)}{2N}}\right]_+^2\right\} \tag{16}$$

*and in particular $\mathbb{P}(B_N) \leq \delta$.*

*Proof.* Suppose the system is stable, i.e., according to Lemma 1 the packet success probability satisfies

$$q > 1 - \frac{1}{\rho(A)^2} \tag{17}$$

The event $A_N$ that Algorithm 1 returns the correct result in this case corresponds to the event

$$A_N = \{\{\gamma_0, \ldots, \gamma_{N-1}\} \in \{0,1\}^N \text{ s.t.} \tag{18}$$

$$\hat{q}_N - \sqrt{\frac{\log(1/\delta)}{2N}} > 1 - \frac{1}{\rho(A)^2}\}. \tag{19}$$

As a result we have that

$$\mathbb{P}(A_N) = 1 - \mathbb{P}\left[\hat{q}_N - \sqrt{\frac{\log(1/\delta)}{2N}} \leq 1 - \frac{1}{\rho(A)^2}\right] \tag{20}$$

Adding and subtracting $q$ at the right hand side we have that

$$\mathbb{P}(A_N) = 1 - \mathbb{P}\left[\hat{q}_N \leq q - \left(q - 1 + \frac{1}{\rho(A)^2} - \sqrt{\frac{\log(1/\delta)}{2N}}\right)\right] \tag{21}$$

The term in the parenthesis can be in general both negative or positive, hence we consider two cases.

Case I: $q - 1 + \frac{1}{\rho(A)^2} - \sqrt{\frac{\log(1/\delta)}{2N}} > 0$. This is the case where the term in the parenthesis in (21) is positive and we can directly apply Hoeffding's inequality (Lemma 2) to get the desired bound (15).

Case II: $q - 1 + \frac{1}{\rho(A)^2} - \sqrt{\frac{\log(1/\delta)}{2N}} \leq 0$. In this case the bound in (15) becomes

$$\mathbb{P}(A_N) \geq 1 - \exp\{-2N\,0\} = 0 \tag{22}$$

which trivially holds.

A symmetric argument verifies the bound (15) when the system is strictly unstable, i.e., when the packet success probability satisfies $q < 1 - \frac{1}{\rho(A)^2}$.

The bound (16) follows with a similar argument. The fact that $\mathbb{P}(B_N) \leq \delta$ holds by lower bounding the absolute value of the stability margin in (16) by 0. □

Some remarks are in order. First, the probability of incorrect answer is always bounded by the desired confidence level $\delta$ by design. This choice is made for safety, i.e., it is very unlikely that the system is unstable and the algorithm incorrectly returns that the system is stable. This probability goes to zero as the number of samples $N$ grows large.

Second, the probability that the algorithm returns the correct answer grows to one as the number of samples grows to infinity. This is expected from the Law of Large Numbers as already mentioned. But for finite number of samples there will be a probability of not being able to determine stability, i.e., the algorithm may return 'undetermined'. More importantly, the probability of correct answer depends on how far the system is from stability, i.e., the term $\left|q - 1 + \frac{1}{\rho(A)^2}\right|$. The largest the stability margin the easier it is to estimate the correct result.

The theorem also assumes that $q \neq 1 - \frac{1}{\rho(A)^2}$, i.e., that there is a non-zero stability margin in view of Lemma 1. Technically the reason is that in that case while $\hat{q}_N$ converges to $q$ it may take values both above and below the limit $q$ and hence the algorithm may oscillate between the answers 'Stable' and 'Unstable' as the number of samples increases.

Using again Hoeffding's inequality we next characterize the sample complexity of the algorithm, i.e., the rate at which the probability of error diminishes as the number of data grows given the system parameters. The following is a direct consequence of the previous Theorem.

**Corollary 1.** *Consider the setup of Theorem 1. If the number of samples satisfies*

$$N \geq \frac{2\log(1/\delta)}{(q - 1 + \frac{1}{\rho(A)^2})^2} \tag{23}$$

*then the procedure correctly determines the stability or instability of the system with probability $(1 - \delta)$.*

The number of samples required depends on the true channel success rate $q$ which is unknown so it is not directly useful but provides intuition. We observe from this result that the sample complexity scales well with the desired confidence level. An order of magnitude improvement in confidence can be guaranteed with just doubling the amount of data samples. On the other hand, the sample complexity does not scale well with the system stability margin. Reducing the stability margin $|q-1+\frac{1}{\rho(A)^2}|$ by a factor of $\beta$ requires $\beta^2$ more channel samples. In particular for a given channel, it becomes easier to verify the stability of a very slow system or a very fast system, otherwise the number of samples grows unbounded at the critical point where the stability margin vanishes.

These observations mean that it becomes impractical to verify stability under all plant and channel configurations, but the above sample complexity can be useful as follows. It describes the amount of channel samples required if we are willing to verify stability with high confidence up to a certain system stability margin.

### B. Control Performance Analysis Using Channel Samples

Beyond verifying stability we are interested in collecting channel samples in order to estimate the control cost of the system over the unknown channel, supposing the system is stable. Formally, given some channel samples we are looking for a high-confidence upper bound on the control cost, i.e., a value $\hat{J}_N$ such that $\mathbb{P}(J(q) \leq \hat{J}_N) \geq 1-\delta$.

The proposed procedure is again based on high confidence bounds on the true channel success rate. Intuitively with the collected samples we can construct a high-confidence lower bound the channel success $q$. Since the function $J(q)$ is non-increasing according to Lemma 1, we can construct a high-confidence upper bound on the control cost by computing the control cost exactly at the lower bound on $q$. This is described again in Alg. 1.

**Theorem 2.** *Consider the switched linear system (1) over an i.i.d. Bernoulli binary channel with an unknown success probability* $q \in [0,1]$ *and assume* $q > 1-\frac{1}{\rho(A)^2}$. *Consider the control cost analysis procedure developed in Algorithm 1 using* $N$ *i.i.d. channel samples drawn with success rate* $q$ *and some parameter* $\delta \in (0,1)$. *If the number of samples is*

$$N \geq \frac{2\log(1/\delta)}{(q-1+\frac{1}{\rho(A)^2})^2} \tag{24}$$

*then the procedure returns an upper bound on the control cost with probability* $(1-\delta)$, *i.e.,*

$$\mathbb{P}(J(q) \leq \hat{J}_N) \geq 1-\delta. \tag{25}$$

*Proof.* Following the same arguments as in the proof of Corollary 1, if the number of samples satisfies (24), then the event $1-\frac{1}{\rho(A)^2} < q_{\min}$ occurs with probability at least $1-\delta$, which means that Algorithm 1 returns a cost value, i.e., it does not return 'Undetermined'.

Consider the if close of Algorithm 1 and note that by construction and using Lemma 1(2) the algorithm returns the value $\hat{J}_N = J(q_{\min})$ where $q_{\min}$ is as computed by the algorithm. Note also by Lemma 1 that the control cost $J(q)$ is non-increasing in the packet success rate $q$. Hence if $q_{\min} \leq q$, then $\hat{J}_N = J(q_{\min}) \geq J(q)$. As a result we have that

$$\mathbb{P}(J(q) \leq \hat{J}_N) \geq \mathbb{P}(q_{\min} \leq q) \tag{26}$$

$$= 1-\mathbb{P}(q_{\min} \geq q) = 1-\mathbb{P}(\hat{q}_N - \sqrt{\frac{\log(1/\delta)}{2N}} \geq q) \tag{27}$$

where the last equality hold by substituting $q_{\min}$ as computed by the algorithm.

We can now employ directly Lemma 3 to verify (25). □

Interestingly the proposed cost performance analysis has the same sample complexity as the stability analysis according to Corollary 1.

**Remark 1.** The assumption that the collected channel samples are i.i.d. following a Bernoulli distribution is crucial in the above results. In practice, only the channel sample data is available and no a priori knowledge about their distribution class, e.g., whether they are i.i.d., as in this paper, or whether they are correlated or even non-stationary. This is a serious practical concern, i.e., our procedure does not necessarily obey the bounds given in Theorem 1 above. Ideally, one should provide a more robust sample-based stability analysis. This is the topic of future work.

## IV. Numerical simulations

We consider a system of the form (1) with spectral radius $\rho(A)=2$ that evolves over a Bernoulli channel with success rate $q=0.9$. For this values the system is stable because (3) holds. For 1000 trials we draw $N=2000$ i.i.d. channel samples according to the success rate $q$. For each trial we run the stability test described in Algorithm 1.

In Fig.2, for different trials we plot the value of the high-confidence lower bound $q_{\min}$ on the true packet success rate $q$ computed by Algorithm 1 as the number of samples $N$ grows. These lower bounds converge to the true packet success, also plotted in the figure. We also plot the minimum packet success rate required for stability which is $1-1/\rho(A)^2$ as described in Lemma 1. Algorithm 1 checks stability by checking whether the lower bounds exceed the minimum packet success rate. As the number of channel samples grows, on average more of the lower bounds exceed the threshold and the algorithm correctly verifies the stability of the system.

We record the responses of the algorithm as 'Unstable', 'Stable', 'Undetermined'. Across all trials we average how many times the algorithm returns the correct answer 'Stable'. This is an empirical evaluation of the correctness of the algorithm, similar to the theoretical bound described by Theorem 1 (cf. (15)). In Fig. 3 we plot both the empirical average correctness of the algorithm as well as its theoretical bound as a function of the number of channel samples drawn. First, we observe that the theoretical bound indeed is a lower bound on the average correctness of the algorithm. We note also that the bound is not tight. That means that fewer channel samples are actually required to learn whether the system is stable or not than what is predicted by our theoretical bound. The reason is that Hoeffding's inequality is conservative, as already mentioned after Lemma 2. Empirically however the rate at which the algorithm correctly learns the system

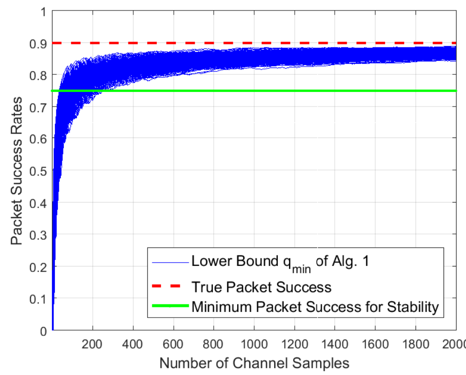


Fig. 2. We consider a system and channel that lead to stability. For different trials we plot the value of the high-confidence lower bound on the true packet success rate $q$ computed by Algorithm 1 as the number of samples $N$ grow. These lower bounds grow on average above the minimum packet success rate required for stability as described in Lemma 1, also plotted. As the number of channel samples grows, on average the algorithm correctly verifies the stability of the system.

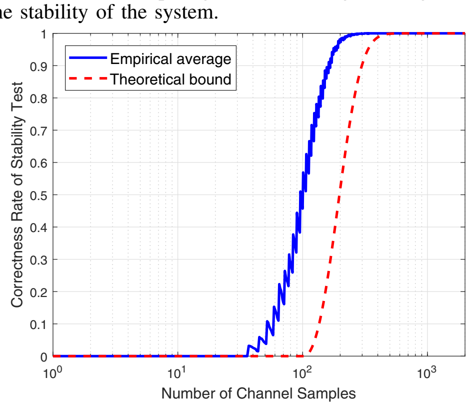


Fig. 3. We consider a system and channel that lead to stability. As the number of channel samples grows, the probability that Algorithm 1 correctly verifies the stability of the system grows. The theoretical bound by Theorem 1 is below the empirical bound obtained by simulation.

stability as the number of samples grows seems to match the rate at which the theoretical bound grows. We note that the spikes appearing in the figure are not noise due to the random samples, they appear because the number of packet successes is a discrete variable instead of continuous.

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