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Scenario-based Model Predictive Control for Energy Harvesting Actuators

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Abstract— **In this paper, we develop a method of control for energy harvesting devices - i.e. systems which can restore their energy reserves online - fit for the purpose of using actuators for tracking a stochastic trajectory. This is a problem of significant novelty, insofar that most prior work concerning energy harvesting devices focuses on energy harvesting communication nodes, and not on devices capable of actuation. Moreover, it is a problem with significant potential applications, as endowing devices with a means for determining how and when energy should be used to best accomplish an assigned task is a central engineering problem faced in developing automation on a large scale. We take a scenario-based model predictive control approach to solving this problem, in which stochastic models of the energy arrival and target evolution processes are used in order to generate a random optimization problem, the solution of which generates a sequence of controls to be applied by the device's actuators. We show that the optimization required by this approach admits a small convex formulation, which may be solved efficiently. We examine the efficacy of the designed controller through numerical simulations.**

I. Introduction

Energy harvesting devices - those systems which are capable of restoring their energy reserves through interacting with their environment - represent an area of significant promise for the development of future technologies. Any applications which involve sensors and robots operating independent of human intervention for long periods of time require that they be able to adequately reason about the availability of energy, both current and future, when planning their actions. Such applications include those as adventurous as space exploration [1], as vital as environmental monitoring [2], and as pragmatic as security surveillance [3]. Despite applications which demand a full understanding of the subject, there are many open questions left to be resolved.

Most work to date concerning the development of mathematics for the optimal use of energy harvesting devices has concerned communication nodes, for which the harvested energy is used to supply power for the transmission of wireless signals. Principally, authors in this area are concerned with maximizing performance of communication networks with respect to communication-specific criteria. A recent review of this area can be found in [4]. Only a few recent works [5]–[7], have studied problems concerning the control performance of a plant coupled to an energy harvesting device, with [5] studying an optimal Linear-Quadratic-Gaussian control problem with feedback coming from an energy-harvesting sensor, [6] demonstrating when estimation error will remain bounded under certain types of sensing policies with simple energy harvesting sources, and [7], [8] analyzing the stability of systems using feedback from energy harvesting sensors.

This work concerns a significantly different problem, that of designing a control method for an energy harvesting device which uses the energy collected from harvesting to supply power to actuators as a means of affecting the plant's state. In this context, we face several difficulties not present in the consideration of energy harvesting communication nodes. Principally, prior works consider cases in which energy is consumed by a sensor node, for the purpose of providing feedback to the plant. In such a framework, the energy consumption of the plant is left unconsidered, which prevents the consideration of many devices, e.g. robots. Our work is the first to address such a problem.

In so doing, we demonstrate that scenario-based random convex programming techniques can be used to define a model predictive control procedure which approximately solves a chance-constrained optimization problem minimizing the extent to which the evolution of the plant deviates from a randomly moving target. While scenario-based methods have been in use in the model predictive control literature for some time (see, e.g., [9]–[11] and the references therein), the problem we address here is itself novel, and is notable insofar that it addresses a system with partially nonlinear dynamics. Moreover, we do so by developing a convex representation that is small, and whose size is invariant to the number of scenarios used to approximate the chance constraints. That is to say, a key technical contribution of this work is a novel convex programming representation of the nonconvex programming problem which arises from the finite horizon optimal control of energy harvesting actuators, which does not generate a large-scale optimization problem when many scenarios are considered. This is critical, as a finite horizon optimal control problem must be solved at each stage during control of a process when applying model predictive control techniques, and its size is a limiting feature present in many applications of scenario-based approximation (see, e.g. [10], [12], [13], wherein the constraint set grows linearly with the number of scenarios).

The paper is organized as follows. In Section II, we define an abstract model of a dynamical energy harvesting device, and state the problem studied in the remainder. In Section III, we formally construct the model predictive control method proposed as a means of controlling a dynamical energy harvesting device. In Section IV, we study an application of this framework, that of controlling the position of a mass actuated by an electric motor so as to follow a stochastic target. We also provide a detailed physical model which confirms that a realistic plant can satisfy the assumptions of our abstract model. Note that where necessary, detailed proofs have been removed from the text to conserve length, but will appear in a later paper.

Notation: We denote by $[k]$ the first k natural numbers, i.e. $[k] = \{1, 2, \dots, k\}$. We denote by $[k]_0$, the union of the first k natural numbers with $\{0\}$, i.e. $[k]_0 = \{0\} \cup [k]$. We denote by $[\![z]\!]_a^b$ the projection of z into the interval $[a, b]$, i.e. $[\![z]\!]_a^b = \min\{\max\{z, a\}, b\}$. We denote by $x_{(j|t)}$, the value

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of the state variable $x(j+t)$, and use similar notation for other quantities which change with respect to time. •

II. Model and Problem Statement

In this section, we formally define the components of the system studied in this paper, as well as our problem statement. Broadly speaking, our system is a standard linear plant augmented with capabilities for explicitly modeling the energy consumed by the system's actuators, and the process by which energy is restored to the system.

A. Plant Model

We consider a linear plant, with discrete-time dynamics

$$x_{t+1} = Ax_t + Bu_t, \tag{1}$$

in which $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times p}$ are real matrices of appropriate dimension, x_t denotes the state of the system at time t, and u_t denotes the control input at time t. At every time t, the plant is supplied with some choice of control u_t, which is an element of some convex set of possible control signals U, which we assume to contain the origin, i.e. we assume that at any time it is permissible to take no action. Here, we assume the set U is a given as part of our problem's abstraction. In practice, it should be chosen so as to adequately model the limitations of the plant's actuators.

B. Energy Model

We adopt the standard model of energy harvesting devices used in the communications community to our setting, making as few of changes as possible. Letting R_t denote the amount of energy reserved in the device's battery for use by the actuator, and H_t denote a random variable enumerating the amount of available energy to be harvested by the system at time t, the process $\{R_t\}$ evolves according to

$$R_{t+1} = [\![R_t + H_t - \mathcal{C}(u_t)]\!]_0^{R_{\text{cap}}}, \tag{2}$$

in which R_{cap} is some finite upper-capacity which limits the battery's storage capabilities, and $\mathcal{C}$ is a function which maps the control input at time t to its energy consumption. We assume $\mathcal{C}$ to be convex and definite with respect to the origin, which we expect to not be a substantial limitation. Indeed, whenever we can verify that the average power consumed by the plant under a particular fixed control input u is convex and definite, e.g. proportional to a norm of the signal, a simple integration argument shows that $\mathcal{C}$ is convex and definite as well. This is precisely the case encountered for a robot which uses DC motors for actuation and uses motor current as a means of control, as we show with respect to the application considered in Section IV.

Because performing actuation requires the consumption of energy, we enforce that at all times, the energy signal is chosen so as to respect the energy causality constraint

$$\mathcal{C}(u_t) \leq R_t \tag{3}$$

at all times. Intuitively speaking, (3) enforces that the plant consumes only the energy made available to it from the system's energy reserve. A key component of our problem is designing a control law for the plant such that it adequately accounts for the future availability of energy, and balances it against the demands placed on it by the controller's objective.

C. Objective Model

We wish for our controller to track a stochastically evolving target. This captures as a special case the problem of regulating a plant so as to maintain a fixed point, while also allowing us to model more complicated situations, such as tasking a robot to follow a randomly moving target.

In order to adequately model this problem, we assume there is some stochastic process $\{q_t\}$ which determines the evolution of the target trajectory, and some other stochastic process $\{Q_t\}$ which determines the evolution of the norm used to penalize deviating from the target trajectory. More concretely, we assume the controller takes as its task the objective of minimizing the cumulative deviation from the target path, starting from the current time, through some fixed lookahead horizon τ. We denote the cumulative deviation from the desired target path over the interval $[t, t+\tau]$, under a fixed control signal as measured by the quadratic norm defined by $\{Q_t\}$ as $D_t(u) \triangleq \sum_{j=0}^{\tau} d_{(j|t)}(u)$, where we define $d_{(j|t)}(u)$ as

$$d_{(j|t)}(u) \triangleq (x_{(j|t)}(u) - q_{(j|t)})^T Q_{(j|t)} (x_{(j|t)}(u) - q_{(j|t)}),$$

where $x_{(j|t)}$ is defined as the value of the state vector process evaluated at time $t+j$, evolving under control signal u, i.e.

$$x_{(j|t)}(u) \triangleq A^j x_{(0|t)} + \sum_{k=0}^{j-1} A^{(j-k)-1} B u_{(k|t)}. \tag{4}$$

Note that in principle, the developments which take place in this paper depend in no way on the particular form of the objective function, other than its convexity. We focus on a stochastically evolving quadratic form here for purposes of simplicity and concreteness, but the results themselves are not limited to this consideration.

D. Problem Statement

In this paper, our core problem is that of considering how to minimize the expected deviation from the target process $\mathbb{E}[D_t(u)]$ over the chosen lookahead horizon τ as a means of recursively generating an input signal for an energy harvesting device's actuators. The principal difficulty we encounter in addressing this problem is due to ensuring that the energy causality constraint (3) is considered in an appropriate manner by the controller. As (3) is defined by an inequality between a function of the control signal and a random variable, this necessarily means that finding an optimal control law would involve solving a constrained stochastic control problem, which is intractable in all but the simplest circumstances (see, e.g., [14] or [15]).

For this reason, we address this task by way of designing a stochastic model predictive controller (SMPC), which guarantees that at each time the designed control signal is energy feasible for all future times over the control horizon with sufficiently large probability. While the design of effective SMPC schemes is itself often a difficult problem (see, e.g. [16] for a recent review), we show in the remainder that the problem specified in this section has enough structure so as to admit a controller which performs well, despite the nonlinearities inherent to the energy reserve dynamics.

III. Controller Design

In this section, we design a stochastic model predictive controller for the problem defined in Section II. Principally, our design technique relies on the use of scenario-based random convex programming as an approximation for a chance-constrained optimization problem. While in general, such techniques result in the generation of large-scale optimization problems [10], [12], [13], and are beholden to the linearity of the system's dynamics in order to generate convex optimization problems, we show that in spite of the nonlinearity of the energy reserve process dynamics (2), the structure of the present problem allows a convex formulation which is size invariant with respect to the number of scenarios considered.

Recalling the notation defined in Section II, we may define a finite-time, chance-constrained optimal control problem (CCOCP) which formalizes the goal of minimizing the expected deviation of the plant from the target as follows

$$\underset{u\in U^{\tau-1}}{\text{minimize}} \quad \mathbb{E}[D_t(u)] \tag{5a}$$

$$\text{subject to } \; x_{(j+|t)} = Ax_{(j|t)} + Bu_{(j|t)}, \tag{5b}$$

$$R_{(j+|t)} = [\![R_{(j|t)} + H_{(j|t)} - \mathcal{C}(u_{(j|t)})]\!]_0^{R_{\text{cap}}}, \tag{5c}$$

$$\mathbb{P}\{\cap_{j=0}^{\tau-1}\{\mathcal{C}(u_{(j|t)}) \leq R_{(j|t)}\}\} \geq 1-\epsilon, \tag{5d}$$

where we note that we need not consider $\{x_{(j|t)}\}_{j=0}^{\tau}$ explicitly as optimization variables, by eliminating them through the relation (4). Note that we force the controller to account for the availability of energy in its decision-making by introducing the chance constraint (5d). Explicitly, (5d) enforces that the designed control signal $u \in U^{\tau-1}$, is feasible for all states reached over the lookahead horizon with probability at least $1-\epsilon$, where ϵ is a free parameter to be tuned by the control designer in service of improving performance.

It is important to note that while the particular choice of ϵ is an adjustable parameter, this constraint is necessary to adequately model the future evolution of the plant. The nature of our system model dictates that at each time t, the control u_t applied to the plant be energy feasible, i.e. obey the constraint (3). If at a particular time t, the implemented u_t is chosen so as to be optimal with respect to planned future controls which are not energy feasible with high probability, it will cause the implemented controls at future times to deviate from the anticipated plan with high probability. As such, we should expect that choosing ϵ too large has the effect of generating controls which are overly-myopic, with performance depending highly on the realizations experienced. If ϵ is chosen too small, we should expect the controller to behave too conservatively, unable to incorporate into its plan even a slight risk that energy might run out. We observe this in the simulations (Section IV).

A. CCOCP for a Fixed Scenario

In this subsection, we show how to transform the CCOCP (5) into a convex problem under a fixed scenario, i.e we consider a deterministic instance of the problem. We do so both to explicitly demonstrate the role uncertainty plays in controlling systems with energy harvesting actuators, and because the fixed-scenario problem is used in Section III-B as a means to approximate the CCOCP (5), even when the problem's stochasticity is not trivial.

Fix some scenario ζ, and let D_t^{ζ} denote the objective function at time t under the scenario ζ, with $H_{(j|t)}^{\zeta}$ defined similarly. In this case, we see that the CCOCP becomes the deterministic optimal control problem

$$\underset{u\in U^{\tau-1}}{\text{minimize}} \quad D_t^{\zeta}(u) \tag{6a}$$

$$\text{subject to } \; x_{(j+|t)} = Ax_{(j|t)} + Bu_{(j|t)}, \tag{6b}$$

$$R_{(j+|t)} = [\![R_{(j|t)} + H_{(j|t)}^{\zeta} - \mathcal{C}(u_{(j|t)})]\!]_0^{R_{\text{cap}}}, \tag{6c}$$

$$\mathcal{C}(u_{(j|t)}) \leq R_{(j|t)}, \tag{6d}$$

which, as stated, is nonconvex, due to the nonlinear dynamics of the energy reserve propagation process $\{R_t\}$, denoted here as (6c). From this, it may at first seem as though the problem is inherently difficult, as the nonconvexity present in (6) may be irremovable. This is not the case, as we now show.

Let $\mathcal{U}_{\zeta}$ denote the feasible set of (6), and with evolution governed by a particular scenario ζ. We now show that $\mathcal{U}_{\zeta}$ is convex for all choices ζ. In particular, the next result shows that enforcing the feasibility constraint $u \in \mathcal{U}_{\zeta}$ is equivalent to enforcing $O(\tau^2)$ convex inequality constraints, in addition to enforcing that $u \in U^{\tau-1}$.

Lemma 1 (Convex Feasible Set Representation) *Let $\mathcal{U}_{\zeta}$ be the set of control signals $u \in U^{\tau-1}$ which are feasible to the fixed scenario optimal control problem* (6) *with scenario ζ. A control signal $u \in U^{\tau-1}$ is in $\mathcal{U}_{\zeta}$ if and only if it satisfies the system of inequalities*

$$\sum_{k=0}^{j} \mathcal{C}(u_{(k|t)}) \leq R_{(0|t)} + \sum_{k=0}^{j-1} H_{(k|t)}^{\zeta}; \tag{7a}$$

$$\sum_{k=k_0}^{j} \mathcal{C}(u_{(k|t)}) \leq R_{\text{cap}} + \sum_{k=k_0}^{j-1} H_{(k|t)}^{\zeta}; \tag{7b}$$

for all $k_0 \in [j]$ and all $j \in [\tau-1]_0$. Moreover, since $\mathcal{C}$ is assumed to be convex, the fixed-uncertainty CCOCP (6) *is a convex optimization problem.*

Note that showing (6) to be convex can be done by way of an alternative argument. If one neglects the non-negativity constraints of the projection defining the dynamics of $\{R_t\}$, one can show that with respect to u, each $R_{(j|t)}$ is a concave function, by noting that it is defined as a minimum of a constant and a concave function of u. In so doing, one may eliminate the nonlinear equality constraints from (6), and add explicit non-negativity constraints for each $R_{(j|t)}$. While this argument is elegant, it suffers from the fact that $R_{(j|t)}$ is a *non-differentiable* function of u regardless of the form of the energy function $\mathcal{C}$, and is indeed an implicit function of ζ as well. Both of these complicate the efficient solution of the resulting optimization problem.

The inequality representation (7) of the feasible set presented in Lemma 1 avoids both drawbacks. It makes clear that the curvature properties of $\mathcal{C}$ are inherited immediately by the feasible set: if $\mathcal{C}$ is an affine function, then the feasible set of (6) is a polyhedron, if $\mathcal{C}$ is a quadratic function, then the feasible set of (6) is an ellipsoid, and so forth. This is important, as optimization over *arbitrary* convex sets can be computationally challenging, especially when the functions defining the constraint sets are non-differentiable, as they would be with the mentioned alternative construction.

Additionally, it provides a representation in which the role of the scenario ζ decouples from the remainder of the problem, a feature we exploit in Section III-B.

B. Scenario-based MPC for EHS

In this subsection, we use the convex representation of the fixed-scenario problem in order to construct means of generating a random convex program which is feasible to the chance-constrained optimization problem (5) with at least a chosen confidence β. To do so, we rely on mathematical results which are well-known, which explore the relation between randomly generated convex programs, and their chance-constrained counterparts [17]. These ideas have been successfully applied to other model predictive control problems in the past, with their primary practical drawback being the complexity introduced when the generated scenarios induce a large set of constraints, as for example in [9], [10], [12], [13]. This is not a feature of the problem we study.

Fixing a particular choice of confidence $\beta \in (0,1)$ and constraint satisfaction $\epsilon \in (0,1)$, a result from the theory of random convex programming [17, Theorem 1] gives that selecting the number of scenarios N_s so as to satisfy

$$\beta \leq \sum_{i=0}^{d-1} \binom{N_s}{i} \epsilon^i (1-\epsilon)^{N_s - i}, \tag{8}$$

ensures that the chance constraint (5d) is satisfied with at least probability $(1-\beta)$, where d is the number of decision variables in the considered problem (here $d = p\tau$). It has also been shown [9, Equation 3] that (8) is implied by the more explicit inequality $N_s \geq \frac{2}{\epsilon}\ln\frac{1}{\beta} + 2d + \frac{2d}{\epsilon}\ln\frac{2}{\epsilon}$, which demonstrates that the sample complexity grows at no worse than a logarithmic rate with respect to β^{-1}, a linear rate with respect to ϵ^{-1}, and a linear rate with respect to d. As such, we may in practice choose β so small as to be very nearly certain that our chance constraint is satisfied.

Given that we have already shown that for each scenario, we can represent the CCOCP as a convex optimization program with $O(\tau^2)$ constraints, it follows immediately that we can construct a random convex program which approximates (5) with $O(\tau^3)$ constraints, for fixed constraint violation ϵ, and confidence β. By explicitly enumerating the generated constraints and taking a sample-average approximation to the objective function, we get the random convex program (RCP)

$$\begin{aligned} \underset{u \in U^{\tau-1}}{\text{minimize}} \quad & \sum_{\zeta \in \mathcal{Z}} D_t^{\zeta}(u) N_s^{-1} \\ \text{subject to} \quad & u \in \cap_{\zeta \in \mathcal{Z}} \{\mathcal{U}_\zeta\}, \end{aligned} \tag{9}$$

where the constraints $u \in \mathcal{U}_\zeta$ are implemented via the representation given in Lemma 1. This, however, is not the best that can be done, as many of the constraints used to represent the intersection in (9) by way of the representation given in Lemma 1 are redundant.

The structure of the problem allows us to compute a more efficient representation by minimizing the right-hand side over the scenario set. This fact is formalized as follows.

Lemma 2 (Reduced Constraint Representation) *Fix some set of sampled scenarios $\mathcal{Z}$. Define the $\mathcal{Z}$-reduced set of control signal constraints $\mathcal{U}_\mathcal{Z}$ as the set of control signals $u \in U^{\tau-1}$ which satisfy*

$$\sum_{k=0}^{j} \mathcal{C}(u_{(k|t)}) \leq R_{(0|t)} + \min_{\zeta \in \mathcal{Z}} \left\{ \sum_{k=0}^{j-1} H^{\zeta}_{(k|t)} \right\}; \tag{10a}$$

$$\sum_{k=k_0}^{j} \mathcal{C}(u_{(k|t)}) \leq R_{\text{cap}} + \min_{\zeta \in \mathcal{Z}} \left\{ \sum_{k=k_0}^{j-1} H^{\zeta}_{(k|t)} \right\}; \tag{10b}$$

for all $k_0 \in [j]_0$, and all $j \in [\tau-1]_0$. A control signal $u \in U^{\tau-1}$ is feasible to (9) if and only if it is in $\mathcal{U}_\mathcal{Z}$.

Note that Lemma 2 follows immediately from Lemma 1, and noting that since the constraints must be valid for all scenarios, they must hold for the minimal values as well. Combining Lemma 2 with earlier discussion immediately results in the following, highlighting our ability to efficiently find controls which are feasible to (5) with high probability.

Theorem 1 (Efficient RCP Relaxation of CCOCP) *Fix some constraint satisfaction probability ϵ and estimation confidence β. Randomly sample a set $\mathcal{Z}$ of N_s independent, identically distributed scenarios ζ, where N_s satisfies (8). The solution of the random convex program*

$$\begin{aligned} \underset{u \in U^{\tau-1}}{\textit{minimize}} \quad & \sum_{\zeta \in \mathcal{Z}} D_t^{\zeta}(u) N_s^{-1} \\ \textit{subject to} \quad & u \in \mathcal{U}_\mathcal{Z}, \end{aligned} \tag{11}$$

where $\mathcal{U}_\mathcal{Z}$ is defined as in Lemma 2, is feasible to the chance-constrained optimal control problem (5) with probability at least $(1-\beta)$.

Note that a key feature of the representation of the optimization problem given in Theorem 1 is that the program is *size invariant* with respect to the number of scenarios considered to approximate the constraints. This is in contrast to many prior uses of scenario-based optimization ideas in model predictive control, wherein the number of constraints often grows linearly with the number of scenarios used [10], [12], [13]. This issue causes many potential applications of scenario-based optimization ideas to generate random convex programs which are large in the context of their application, and as such prevent their efficient use. It is a remarkable feature of the problem at hand that no such difficulties are encountered. However, it is not unique to this setting, as a similar feature has been shown for stochastic model predictive control with plants subject to stochastic disturbances, with chance constraints enforcing the state to remain in a polytope [18]. The key feature needed to arrive at such a representation is an efficient method for identifying redundant constraints, given a set of sampled scenarios.

Theorem 1 provides us with a road-map for the efficient implementation of a stochastic model predictive controller which addresses the task stated in Section II. Specifically, at each time t, we observe the plant state x_t and the energy reserve state R_t, and generate a set of scenarios $\mathcal{Z}$ from the oracles simulating the target and energy harvesting processes. Then, we generate the reduced RCP (11), solve it, and apply the input corresponding to the current time. As U is assumed to contain the origin, and $\mathcal{C}$ is assumed to be definite with respect to the origin, it follows that (11) is feasible, no matter the states of x_t and R_t. We see in Section IV that applying this procedure produces good results, provided the constraint violation probability (equivalently, the number of scenarios used to approximate the chance constraints) is tuned well.

IV. Simulated Application

In this section, we develop a detailed simulation of an example application. Specifically, we consider the problem of actuating a mass with a DC motor, by way of constant current control, with energy arriving in the system's battery by way of a periodic stochastic source. Abstractly, this can be considered as a generic model for a robot operating in a remote environment, recharging via a solar cell, tasked with regulating its position. Concretely, such a situation may well be encountered by a mobile sensor, which needs to move in order to better observe a moving object, e.g. a wild animal.

A. Plant Modeling

We study the case of a robot with linear dynamics, which are expressed by the following dynamical relation:

$$M\ddot{y} + F\dot{y} = K\vec{i}, \tag{12}$$

in which M is a mass matrix, which models the inertia of the mechanical part of the system, F is a drag coefficient matrix, which models the effect of friction on the mechanical part of the system, and K is a matrix of motor characteristics, which map the current through the motor to the force vector being applied to the system. Note that the dynamics expressed by (12) can be seen as following directly from balancing the forces applied by the motor, given by $K\vec{i}$, against the acceleration force $M\ddot{y}$, and the drag force $F\dot{y}$.

Solving (12) for dynamics on the plant state x when the motor current vector $\vec{i}$ is taken to be the control signal, we see that the evolution of $x = [\dot{y}^T, y^T]^T$ is described by

$$\frac{d}{dt}\begin{bmatrix}\dot{y}\\ y\end{bmatrix} = \begin{bmatrix}-M^{-1}F & 0\\ I & 0\end{bmatrix}\begin{bmatrix}\dot{y}\\ y\end{bmatrix} + \begin{bmatrix}M^{-1}K\\ 0\end{bmatrix}u, \tag{13}$$

where we have implicitly assumed that M is invertible. By fixing a particular control u and integrating (13) forward for Δt seconds, we see that $x(\Delta t)$ satisfies

$$x(\Delta t) = \mathrm{e}^{A_c\Delta t}x(0) + \int_0^{\Delta t}\mathrm{e}^{A_c(\Delta t-\sigma)}B_c u\,\mathrm{d}\sigma, \tag{14}$$

where we have defined

$$A_c = \triangleq \begin{bmatrix}-M^{-1}F & 0\\ I & 0\end{bmatrix},\ B_c \triangleq \begin{bmatrix}M^{-1}K\\ 0\end{bmatrix},$$

and the exponential is the matrix exponential function (see, e.g., [19, Lecture 6]). Consequently, the discrete-time system $x^+ = Ax + Bu$, with $A \triangleq \mathrm{e}^{A_c\Delta t}, B \triangleq \int_0^{\Delta t}\mathrm{e}^{A_c(\Delta t-\sigma)}B_c\mathrm{d}\sigma$, matches the continuous-time system exactly, where time is incremented by Δt units per step.

We study the case in which the motors are supplied by a constant-voltage battery, and as such, basic circuit theory tells us that the power consumed by the motor at time t when driven by a current u_t is given by the expression $P(t) = V_B\|u_t\|_1$, where V_B is the electrical potential of the battery, assumed to be constant. Integrating the power of the control signal over the control time yields the energy cost

$$\mathcal{C}(u) \triangleq \int_0^{\Delta t} V_B\|u(\sigma)\|_1 d\sigma = \Delta t V_B\|u\|_1. \tag{15}$$

As all norms are convex functions and definite with respect to the origin, it follows that $\mathcal{C}$ is a convex function of u, and is definite with respect to the origin. Additionally, since constraints involving non-negative combinations of one 1-norms can be efficiently represented as small systems of affine constraints, the random convex programs required for implementation can be represented as convex quadratic optimization problems with affine constraints, which admit several efficient solution algorithms [20].

This derivation highlights an important general principle. Whenever the map which takes the control signal to the power consumed is convex and definite, the energy function $\mathcal{C}$ is convex and definite as well. We expect that this relation will hold for a wide variety of systems, though verifying this explicitly in diverse contexts is left as a task for future work.

B. Harvest Source Modeling

We assume the plant is supplied with energy from a stochastic, periodic source, which is a model for charging from a solar cell. Letting S denote the maximum charging amount achieved by the system (say, at solar noon), T denote the number of control intervals considered over the course of a day, and L_t be the state of the cloud loss process taking values on some subset of the positive integers $\mathcal{L} = [k]$, the dynamics of $\{H_t\}$ satisfy

$$H_t = S\left[\!\!\left[\sin\left(\frac{2\pi t}{T}\right) - \frac{(L_t-1)}{k-1}\right]\!\!\right]_0^\infty. \tag{16}$$

Note that the the assumption that solar intensity varies as a trigonmetric function of time is based on established models [21], where the particular form of the function depends on the time of year, and location where the intensity is to be measured. The simulations we show here in this paper consider the case in which $S = 0.01$, $T = 48$, and $\{L_t\}$ has four states, with the transition matrix

$$L = \begin{bmatrix}0.30 & 0.35 & 0.00 & 0.00\\ 0.70 & 0.30 & 0.35 & 0.00\\ 0.00 & 0.35 & 0.30 & 0.70\\ 0.00 & 0.00 & 0.35 & 0.30\end{bmatrix}.$$

C. Application Simulation Results

We now evaluate the performance of the proposed control framework with respect to an example system, by way of numerical simulation. For simplicity, we focus here on the case in which the mass is moving along a one-dimensional track, and is moved by a single DC motor, which provides force for the purposes of movement in both directions along the track. As a practical example of such a system, we may think of a camera moving along a track so as to adequately surveil a large area for the presence of intruders.

We take the mass of the system to be 5 kilograms, the drag coefficient to be 10^{-3} Newtons per meter per second, and the torque constant of the motor to be 10^{-1} Newtons per ampere. We allow the motor current to vary from -10 milliamperes to 10 milliamperes. We assume the battery supplying energy to the motor to be operating at a constant electrical potential of 12 volts. Evaluating the expressions derived in Section IV-A for (A, B) using MATLAB's built-in `c2d` function with $\Delta t = 30$ minutes, we arrive at the system model

$$A = \begin{bmatrix}0.700 & 0\\ 1,512 & 1\end{bmatrix},\ \ B = \begin{bmatrix}0.03\\ 28.84\end{bmatrix}, \mathcal{C}(u) = 21.60\|u\|_1,$$

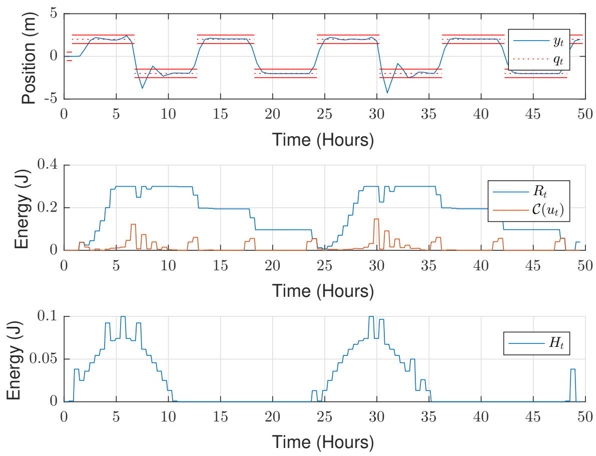

Fig. 1: A sample path of the realized controller performance. The target process is given by a red dashed line, with solid red lines giving a visual reference for a cost penalty of one unit on either side of the target. Notably, we see that the controller chooses to save energy during the day so as to be able to use it to reach the target at night. Hence, the designed controller behaves intelligently with respect to the nature of the energy source.

where the control signal u is the motor current measured in milliamperes, and the energy function is measured in joules. For simplicity, we consider a target that jumps deterministically from the position 2 to the position -2 every 6 hours.

Figure 1 reports the results of the simulated system with a lookahead of 48 time steps, i.e. 24 hours, and in which $3,100$ scenarios are used to approximate the chance constraints, corresponding to a choice of 0.1 probability of chance constraint violation with confidence $\beta = 10^{-3}$. Note that the number of scenarios was tuned in order to optimize performance, and that $3,100$ was an intermediate value among those tested. While defining a formal method for tuning the performance of the controller is left for future work, we expect optimal values to moderate, achieving a balance between aggression and conservatism. We see that when properly tuned, the controller is intelligent enough so as to avoid attempting to match the target trajectory precisely during the day so as to conserve energy for moving at night, when energy is scarce.

V. Conclusions and Future Work

We have presented a model predictive control method for computing controls for the actuators of energy harvesting devices. The method presented incorporates knowledge of a model for how energy arrives in order to inform the design of controls, by way of approximating a chance constraint enforcing the energy feasibility of the signal. Importantly, it was shown that the structure of the problem studied here allows for an optimization formulation whose constraint set is size invariant with respect to the number of scenarios used, which enables its efficient solution.

There are several avenues for interesting future work. The material presented here only covers the case in which the system's plant follows standard linear dynamics, however many interesting systems are inherently nonlinear, and as such, accommodations for such situations must be made, even if only in an *ad hoc* way. The number of scenarios used to approximate the constraints was chosen here in order to guarantee the satisfaction of a chance constraint with high probability. However, it may well be the case that tuning the number of scenarios considered may be best done in order to optimize closed-loop system performance in terms of a specified objective. Such tuning seems entirely possible, but falls outside of the scope of the work considered here.

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