LQG Control and Sensing Co-Design

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***Abstract*—We investigate a linear-quadratic-Gaussian (LQG) control and sensing codesign problem, where one jointly designs sensing and control policies. We focus on the realistic case where the sensing design is selected among a finite set of available sensors, where each sensor is associated with a different cost (e.g., power consumption). We consider two dual problem instances: sensing-constrained LQG control, where one maximizes a control performance subject to a sensor cost budget, and minimum-sensing LQG control, where one minimizes a sensor cost subject to performance constraints. We prove that no polynomial time algorithm guarantees across all problem instances a constant approximation factor from the optimal. Nonetheless, we present the first polynomial time algorithms with per-instance suboptimality guarantees. To this end, we leverage a separation principle, which partially decouples the design of sensing and control. Then, we frame LQG codesign as the optimization of approximately supermodular set functions; we develop novel algorithms to solve the problems; and we prove original results on the performance of the algorithms and establish connections between their suboptimality and control-theoretic quantities. We conclude the article by discussing two applications, namely, *sensing-constrained formation control* and *resource-constrained robot navigation*.**

***Index Terms*—Aerospace engineering, autonomous systems, algorithm design and analysis, computational complexity, multiagent systems, resource management.**

I. Introduction

TRADITIONAL approaches to systems control assume the choice of sensors fixed [1]. The sensors usually result from a preliminary design phase in which an expert selects a suitable sensor suite that accommodates estimation requirements and system constraints (e.g., power consumption). However, the control applications of the Internet of Things (IoT) and Battlefield Things (IoBT) [2] pose serious limitations to the applicability of this traditional paradigm. Now, systems are not designed from scratch; instead, existing, standardized systems come together, along with their sensors, to form heterogeneous teams (such as robot teams), tasked with various control goals: from collaborative object manipulation to formation control [3], [4]. In such heterogeneous networked systems, where new nodes are continuously added and removed from the network, sensor redundancies are created, depending on the task at hand. At the same time, power, bandwidth, and/or computation constraints limit which sensors can be active [5]–[7]. Therefore, to optimize the network's operability and prolong its operation, one needs to decide which sensors are important for the task, and activate only these [7]. Evidently, in large-scale networks, a manual activation policy is not scalable. Thus, ones needs to develop automated approaches. Motivated by this need, we consider the codesign of linear-quadratic-Gaussian (LQG) control and sensor selection subject to sensor activation constraints. Particularly, we assume that the sensor constraints are captured by a prescribed budget (e.g., available battery power), and that each sensor is associated with an activation cost (e.g., power consumption).

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A. Related Work in Control

Traditionally, the control literature has focused on codesigning control, estimation, actuation (i.e., actuator selection), and sensing (i.e., sensor selection) [1], [5]–[41]. However, the focus so far has mostly been different from the codesign problem we consider in this article.

a) The authors in [1] and [8]–[14] assume all sensors are given and active (instead of choosing a few sensors to activate). They focus on the codesign of control and estimation over band-limited communication channels, and investigate tradeoffs between communication constraints (e.g., quantization), and control performance (e.g., stability). In more detail, they provide results on the impact of quantization [10], and of finite data rates [11], [12], as well as, on separation principles for LQG design with communication constraints [13]. Recent works also focus on privacy constraints [14]. For a comprehensive review on LQG control and estimation, we refer to [1], [8], and [9].

b) The authors in [15]–[21] extend the focus of the above-mentioned works, by focusing on the codesign of control, estimation, and sensing. Yet, the choice of each sensor can be arbitrary (instead, in our framework, a few sensors are activated from a given finite set of available ones). For example, the authors in [15]–[19] and [42] propose the optimization of steady-state LQG costs, subject to sparsity constraints on the sensor matrices and/or on the feedback control and estimation gains. Finally, the authors in [20] and [21] augment the LQG cost with an information-theoretic regularizer and design the sensors matrices using semidefinite programming.

c) The authors in [5]–[7], [22]–[41] focus on sensor selection, but they do not consider control aspects (with the exception of [7], [40], and [41], which we discuss

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below). Specifically, the authors in [22] studies sensor placement to optimize maximum likelihood estimation over static parameters, whereas [5], [6], [23]–[27] focus on optimizing Kalman filtering and batch estimation accuracy over nonstatic parameters. The authors in [28], [29], and [31]–[37] present sensor and actuator selection algorithms to optimize the average observability and controllability of systems; the authors in [30] focus on actuator placement for stability in uncertain systems. For additional relevant applications, we refer to [38]. The author in [42, Ch. 6.1.3] focuses on selecting a sensor for each edge of a consensus-type system for $\mathcal{H}_2$ optimization subject to sensor cost constraints and sensor noise considerations (instead, we consider general systems). The author in [39] select the location of a phasor measurement unit (PMU) on a single edge of an electrical network to minimize estimation error (each placement happens independently of the rest). The authors in [40] and [41] study sensor placement to optimize a steady-state LQG cost; although the latter case is similar to our framework (we optimize a finite horizon LQG cost, instead of a steady state), the authors focus only on a small-scale system with a few sensors, where a brute-force selection is viable, and no scalable algorithms are proposed (instead, our focus is on scalable approximation algorithms). Finally, the authors in [7] study an LQG control and scheduling codesign problem, where *decoupled* systems share a wireless sensor network, while power consumption constraints must be satisfied. Instead, we consider coupled systems, a framework that makes our codesign problem inapproximable in polynomial time, in contrast to [7]'s, which is optimally solved in polynomial time.

B. Related Work on Set Function Optimization

In this article, a few sensors must be activated among a set of available ones. This is a combinatorial problem, and we prove it inapproximable: across all problem instances, no polynomial time algorithm can guarantee a constant approximation factor from the optimal. Thus, to provide efficient algorithms with per-instance suboptimality bounds instead, we resort to tools from *combinatorial* optimization, which has been a successful paradigm on this front [38], [38], [43]–[46]. Specifically, the literature on combinatorial optimization includes investigation into (i) *supermodular* optimization subject to *cardinality* constraints (where only a prescribed number of sensors can be active) [47], [48]; (ii) *supermodular* optimization subject to *cost* constraints [49]–[51] (where only sensor combinations that meet a prescribed budget can be active—each sensor has a potentially different activation cost); and (iii) *approximately supermodular* optimization subject to *cardinality* constraints [43]–[46]. The literature does not cover *approximately submodular* optimization subject to *cost* constraints, which is the setup of interest in this article; hence, we herein develop algorithms and novel suboptimality bounds for this case.[1]

[1]The transition from cardinality to cost constraints, in terms of providing efficient algorithms with provable suboptimality bounds, is nontrivial, as it is observed by comparing the widely different proof techniques in [52] for the cardinality case, versus those considered in this article, for the cost case.

C. Contributions to Control Theory

We address an *LQG control and sensing codesign* problem. The problem extends LQG control to the case where, besides designing an optimal controller and estimator, one has to decide which sensors to activate, due to sensor cost constraints and a limited budget. That is, the sensor choice is restricted to a finite selection from the available sensors, rather than being arbitrary (for arbitrary sensing design, see [15]–[18], [20]). In addition, each sensor has a cost that captures the penalty incurred for using the sensor. Since different sensors (e.g., lidars, radars, cameras, lasers) have different power consumption, bandwidth utilization, and/or monetary value, we allow each sensor to have a different cost. We formulate two dual instances of the LQG codesign problem. The first, *sensing-constrained LQG control*, involves the joint design of control and sensing to minimize the LQG cost subject to a sensor cost budget. The second, *minimum-sensing LQG control*, involves the joint design of control and sensing to minimize the cost of the activated sensors subject to a desired LQG cost.

To solve the proposed LQG problems, we first leverage a separation principle that partially decouples the control and sensor selection.[2] As a negative result, we prove that the optimal sensor selection is inapproximable in polynomial time by a constant suboptimality bound across all problem instances. Therefore, we develop algorithms with per-instance suboptimality bounds instead. Particularly, we frame the sensor selection as the optimization of *approximately* supermodular set functions, using the notion of the supermodularity ratio introduced in [53] (see also [44]).[3] Then, we provide the first polynomial time algorithms, which provably retrieve a close-to-optimal choice of sensors, and the corresponding optimal control policy. Specifically, the suboptimality gaps of the algorithms depend on the supermodularity ratio γ of the LQG cost, and we establish connections between γ and control-theoretic quantities, providing computable lower bounds for γ.

D. Contributions to Set Function Optimization

To prove the aforementioned results, we extend the literature on supermodular optimization. Particularly, we provide the first efficient algorithm for approximately supermodular optimization (e.g., LQG cost optimization) subject to *cost* constraints for subset selection (e.g., sensor selection). To this end, we use the algorithm in [51], proposed for *exactly* supermodular optimization, and prove it maintains provable suboptimality bounds for even *approximately* supermodular optimization. Importantly, our bounds improve the previously known bounds for *exactly* supermodular optimization: our bounds become $1-1/e$ for supermodular optimization, tightening the known $1/2(1-1/e)$ [51]. Noticeably, $1-1/e$ is the best possible bound in polynomial time for the supermodular optimization subject to cardinality constraints [57]. That way, our analysis equates the approximation difficulty of *cost* and *cardinality* constrained optimization for the first time (among all algorithms with at most quadratic running time).[4] That way, our results are relevant beyond sensing in control, such as in cost-effective outbreak detection in networks [62].

Similarly, we provide the first algorithm for a minimal cost subset selection subject to desired bounds on an approximately supermodular function. The algorithm relies on a simplification of the algorithm in [51]. Leveraging our novel bounds, we show the algorithm is the first with provable suboptimality bounds given approximately supermodular functions. Notably, for exactly supermodular functions, the bound recovers the well-known bound for cardinality minimization [48];

[2]The separation between control and *sensor selection* is proved with the same steps as the separation of control and *estimation* in standard LQG control theory; e.g., see [20, proof of Lemma 1].

[3]The notion has met already increasing interest in the signal processing and control literature; see, for example, [26], [45], [46], [54]–[56].

[4]Other algorithms, that either achieve the $1-1/e$ bound but are slower ($O(n^5)$ instead of $O(n^2)$ that ours is), or they achieve looser bounds with the same running time, such as the $1-1/\sqrt{e}$, are found in [51], [50], and [58]–[61].

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that way, similarly to above, our analysis equates the approximation difficulty of cost and cardinality minimization for the first time.

E. Application Examples

We demonstrate the effectiveness of the proposed algorithms in numerical experiments, by considering two application scenarios: *sensing-constrained formation control* and *resource-constrained robot navigation*. We present a Monte Carlo analysis for both, which demonstrates that (i) the proposed sensor selection strategy is near-optimal, and, particularly, the resulting LQG cost matches the optimal selection in all tested instances for which the optimal selection could be computed via a brute-force approach; (ii) a more naive selection that attempts to minimize the state estimation error [24] (rather than the LQG cost) has degraded LQG performance, often comparable to a random selection; and (iii) the selection of a small subset of sensors using the proposed algorithms ensures an LQG cost that is close to the one obtained by using all available sensors, hence, providing an effective alternative for control under sensing constraints.

F. Comparison With the Preliminary Results in [52] (Which Coincides With the Preprint [63]).

This article (which coincides with the preprint [64]) extends the preliminary results in [52] and provides comprehensive presentation of the LQG codesign problem, by including both the *sensing-constrained LQG control* (introduced in [52]) and the *minimum-sensing LQG control* problem (not previously published). Moreover, we generalize the setup in [52] to account for any sensor costs (in [52] each sensor has a unit cost, whereas herein sensors have different costs). Also, we extend the numerical analysis accordingly. Moreover, we prove the inapproximability of the problem. Most of the technical results (Theorems 1–4, and Algorithms 2–4) are novel and have not been published.

Organization of the rest of this article. Section II formulates the LQG control and sensing codesign problems. Section III presents a separation principle, the inapproximability theorem, and introduces the algorithms for the codesign problems. Section IV characterizes the performance of the algorithms and establishes connections between their suboptimality bounds and control-theoretic quantities. Section V presents two examples of the codesign problems. Section VI concludes the article. *All proofs are given in the Appendix.*

Notation. Lowercase letters denote vectors and scalars (e.g., v); uppercase letters denote matrices (e.g., M). Calligraphic fonts denote sets (e.g., $\mathcal{S}$). $\mathbf{I}$ denotes the identity matrix.

II. Problem Formulation: LQG Control and Sensing Codesign

Here, we formalize the LQG control and sensing codesign problem considered in this article. Specifically, we present two "dual" statements of the problem: the *sensing-constrained LQG control* and the *minimum-sensing LQG control*.

A. System, Sensors, and Control Policies

We start by introducing the article's framework.

a) System: We consider a discrete-time time-varying linear system with additive Gaussian noise as

$$x_{t+1} = A_t x_t + B_t u_t + w_t, \quad t = 1, 2, \dots, T \tag{1}$$

where $x_t \in \mathbb{R}^n$ is the system's state at time t, $u_t \in \mathbb{R}^{m_t}$ is the control action, w_t is the process noise, A_t and B_t are known matrices, and T is a finite horizon. Also, x_1 is a Gaussian random variable with covariance $\Sigma_{1|1}$, and w_t is a Gaussian random variable with mean zero and covariance W_t, such that w_t is independent of x_1 and $w_{t'}$ for all $t' = 1, 2, \dots, T$, $t' \neq t$.

b) Sensors: We consider the availability of a (potentially large) set $\mathcal{V}$ of sensors, which can take noisy linear observations of the system's state. Particularly,

$$y_{i,t} = C_{i,t} x_t + v_{i,t}, \quad i \in \mathcal{V} \tag{2}$$

where $y_{i,t} \in \mathbb{R}^{p_{i,t}}$ is the measurement of sensor i at time t, $C_{i,t}$ is a sensing matrix, and $v_{i,t}$ is the measurement noise. We assume $v_{i,t}$ to be a Gaussian random variable with mean zero and positive definite covariance $V_{i,t}$, such that $v_{i,t}$ is independent of x_1, and of $w_{t'}$ for any $t' \neq t$, and independent of $v_{i,t't'}$ for all $t' \neq t$, and any $i' \in \mathcal{V}$, $i' \neq i$.

When only a subset $\mathcal{S} \subseteq \mathcal{V}$ of the sensors is active for all $t = 1, 2, \dots, T$, then the measurement model becomes

$$y_t(\mathcal{S}) = C_t(\mathcal{S}) x_t + v_t(\mathcal{S}) \tag{3}$$

where $y_t(\mathcal{S}) \triangleq [y_{i_1,t}^\mathsf{T}, y_{i_2,t}^\mathsf{T}, \dots, y_{i_{|\mathcal{S}|},t}^\mathsf{T}]^\mathsf{T}$, $C_t(\mathcal{S}) \triangleq [C_{i_1,t}^\mathsf{T}, \dots, C_{i_{|\mathcal{S}|},t}^\mathsf{T}]^\mathsf{T}$, and $v_t(\mathcal{S})$ is a mean zero Gaussian noise with cova- riance $V_t(\mathcal{S}) \triangleq \operatorname{diag}(V_{i_1,t}, \dots, V_{i_{|\mathcal{S}|},t})$.

Each sensor is associated with a (possibly different) cost, which captures, for example, the sensor's monetary cost, its power consumption, or its bandwidth utilization. Specifically, we denote the *cost of sensor* i by $c(i) \geq 0$; and the *cost of a sensor set* $\mathcal{S}$ by $c(\mathcal{S})$, where we set

$$c(\mathcal{S}) \triangleq \sum_{i \in \mathcal{S}} c(i). \tag{4}$$

c) Control policies: We consider control policies u_t informed only by the active sensor set $\mathcal{S}$:

$$u_t = u_t(\mathcal{S}) = t u_t\left(y_1(\mathcal{S}), y_2(\mathcal{S}), \dots, y_t(\mathcal{S})\right), \quad t = 1, 2, \dots, T.$$

B. LQG Codesign Problems

We define two versions of the codesign problem: Sensing-constrained LQG control and minimum-sensing LQG control. Their unifying goal is to find active sensors $\mathcal{S}$ and a policy $u_{1:T}(\mathcal{S}) \triangleq \{u_1(\mathcal{S}), u_2(\mathcal{S}), \dots, u_T(\mathcal{S})\}$, such that the sensor cost $c(\mathcal{S})$ is low and the finite horizon LQG cost $h(\mathcal{S}, u_{1:T}(\mathcal{S}))$ below is optimized:

$$h(\mathcal{S}, u_{1:T}(\mathcal{S})) \triangleq \sum_{t=1}^{T} \mathbb{E}\left[\|x_{t+1}(\mathcal{S})\|_{Q_t}^2 + \|u_t(\mathcal{S})\|_{R_t}^2\right] \tag{5}$$

where $Q_1, \dots, Q_T$ are known positive semidefinite matricies, $R_1, \dots, R_T$ are known positive definite matricies, and the expectation is taken with respect to $x_1, w_{1:T}$, and $v_{1:T}(\mathcal{S})$. Particularly, the *sensing-constrained LQG control* minimizes the LQG cost subject to a sensor cost budget, and the dual *minimum-sensing LQG control* minimizes the sensor cost subject to a desired LQG cost.

Problem 1 (Sensing-constrained LQG control): Given a budget $b \geq 0$ on the sensor cost, find sensors $\mathcal{S}$ and a policy $u_{1:T}(\mathcal{S}) \triangleq \{u_1(\mathcal{S}), u_2(\mathcal{S}), \dots, u_T(\mathcal{S})\}$ to minimize the finite horizon LQG cost $h(\mathcal{S}, u_{1:T}(\mathcal{S}))$:

$$\min_{\substack{\mathcal{S} \subseteq \mathcal{V} \\ u_{1:T}(\mathcal{S}).}} \; h(\mathcal{S}, u_{1:T}(\mathcal{S})), \;\; \text{s.t.} \;\; c(\mathcal{S}) \leq b. \tag{6}$$

Problem 1 models the practical case where we cannot activate all sensors (due to power, cost, or bandwidth constraints), and instead need to activate a few sensors to optimize control performance. If the budget is increased so all sensors can be active, then Problem 1 reduces to standard LQG control.

Problem 2 (Minimum-sensing LQG control): Find a minimal cost sensor set $\mathcal{S}$, and a policy $u_{1:T}(\mathcal{S})$, such that the finite horizon LQG cost $h(\mathcal{S}, u_{1:T}(\mathcal{S}))$ is at most κ, where $\kappa \geq 0$ is given as

$$\min_{\substack{\mathcal{S} \subseteq \mathcal{V}, \\ u_{1:T}(\mathcal{S}).}} \; c(\mathcal{S}), \;\; \text{s.t.} \;\; h(\mathcal{S}, u_{1:T}(\mathcal{S})) \leq \kappa. \tag{7}$$

Problem 2 models the practical case where one wants to design a system with a prescribed performance, while incurring in the smallest sensor cost.

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Remark 1 (Case of uniform-cost sensors): When all sensors $i \in \mathcal{V}$ have the same cost, say $c(i) = \bar{c} > 0$, the sensor budget constraint becomes a cardinality constraint

$$c(\mathcal{S}) \leq b \;\Leftrightarrow\; \sum_{i\in\mathcal{S}} c(i) \leq b \;\Leftrightarrow\; |\mathcal{S}|\bar{c} \leq b \;\Leftrightarrow\; |\mathcal{S}| \leq \frac{b}{\bar{c}}. \quad (8)$$

III. Codesign Principles, Hardness, and Algorithms

We leverage a separation principle to derive that the optimization of the sensor set $\mathcal{S}$ and of the control policy $u_{1:T}(\mathcal{S})$ can happen in cascade. However, we show that optimizing for $\mathcal{S}$ is inapproximable in polynomial time. Nonetheless, we then present polynomial time algorithms for Problems 1 and 2 with provable per-instance suboptimality bounds. Particularly, the bounds are presented in Section IV.[5]

A. Separability of Optimal Sensing and Control Design

We characterize the jointly optimal control and sensing solutions for Problems 1 and 2, and prove they can be found in two separate steps, where first the sensor set is found, and then the control policy is computed.

Theorem 1 (Separability of optimal sensor set and control policy design): For any active sensor set $\mathcal{S}$, let $\hat{x}_t(\mathcal{S})$ be the Kalman estimator of the state x_t, and $\Sigma_{t|t}(\mathcal{S})$ be $\hat{x}_t(\mathcal{S})$'s error covariance. Additionally, let the matrices Θ_t and K_t be the solution of the following backward Riccati recursion:

$$\begin{aligned} S_t &= Q_t + N_{t+1} \\ N_t &= A_t^{\mathsf{T}}(S_t^{-1} + B_t R_t^{-1} B_t^{\mathsf{T}})^{-1} A_t \\ M_t &= B_t^{\mathsf{T}} S_t B_t + R_t \\ K_t &= -M_t^{-1} B_t^{\mathsf{T}} S_t A_t \\ \Theta_t &= K_t^{\mathsf{T}} M_t K_t \end{aligned} \quad (9)$$

with boundary condition $N_{T+1} = 0$.

1) *(Separability in Problem 1)* Any optimal solution $(\mathcal{S}^\star, u^\star_{1:T})$ to Problem 1 can be computed in cascade:

$$\mathcal{S}^\star \in \arg\min_{\mathcal{S}\subseteq\mathcal{V}} \sum_{t=1}^{T} \mathrm{tr}[\Theta_t \Sigma_{t|t}(\mathcal{S})], \;\; \text{s.t.} \;\; c(\mathcal{S}) \leq b \quad (10)$$

$$u_t^\star = K_t \hat{x}_t(\mathcal{S}^\star), \quad t = 1, \ldots, T. \quad (11)$$

2) *(Separability in Problem 2)* Define the constant $\bar{\kappa} \triangleq \kappa - \mathrm{tr}(\Sigma_{1|1} N_1) - \sum_{t=1}^{T} \mathrm{tr}(W_t S_t)$. Any optimal solution $(\mathcal{S}^\star, u^\star_{1:T})$ to Problem 2 can be computed in cascade:

$$\mathcal{S}^\star \in \arg\min_{\mathcal{S}\subseteq\mathcal{V}} c(\mathcal{S}), \;\; \text{s.t.} \;\; \sum_{t=1}^{T} \mathrm{tr}[\Theta_t \Sigma_{t|t}(\mathcal{S})] \leq \bar{\kappa} \quad (12)$$

$$u_t^\star = K_t \hat{x}_t(\mathcal{S}^\star), \quad t = 1, \ldots, T. \quad (13)$$

Remark 2 (Certainty equivalence principle): The control gain matrices $K_1, K_2, \ldots, K_T$ are the same as the ones that make the controllers $(K_1 x_1, K_1 x_2, \ldots, K_T x_T)$ optimal for the perfect state-information version of Problem 1, where the state x_t is known to the controllers [1, Ch. 4].

Theorem 1 decouples the sensing design from the control policy design. Particularly, once an optimal sensor set $\mathcal{S}^\star$ is found, then the optimal controllers are equal to $K_t \hat{x}_t(\mathcal{S}^\star)$, which correspond to the standard LQG control policy.

An intuitive interpretation of the sensor design steps in (10) and (12) follows next.

[5]The novelty of the algorithms is also discussed in Section IV.

Algorithm 1: Joint Sensing and Control Design for Problem 1.

Input: Horizon T; system in (1); covariance $\Sigma_{1|1}$; LQG cost matrices Q_t and R_t in (5); sensors in (2); sensor budget b; sensor cost $c(i)$, for all $i \in \mathcal{V}$.

Output: Active sensors $\widehat{\mathcal{S}}$, and controls $\hat{u}_1, \hat{u}_2, \ldots, \hat{u}_T$.

1: Compute $\Theta_1, \Theta_2, \ldots, \Theta_T$ using (9).
2: Return $\widehat{\mathcal{S}}$ returned by Algorithm 2, which finds a solution to the optimization problem in (10);
3: Compute $K_1, K_2, \ldots, K_T$ using (9).
4: At each $t = 1 \ldots, T$, compute the Kalman estimate of x_t:

$$\hat{x}_t \triangleq \mathbb{E}[x_t | y_1(\widehat{\mathcal{S}}), y_2(\widehat{\mathcal{S}}), \ldots, y_t(\widehat{\mathcal{S}})];$$

5: At each $t = 1, \ldots, T$, return $\hat{u}_t = K_t \hat{x}_t$.

Remark 3 (Control-aware sensor design): To provide insight on $\sum_{t=1}^{T} \mathrm{tr}[\Theta_t \Sigma_{t|t}(\mathcal{S})]$ in (10) and (12), we rewrite it as

$$\begin{aligned} \sum_{t=1}^{T} \mathrm{tr}[\Theta_t \Sigma_{t|t}(\mathcal{S})] &= \sum_{t=1}^{T} \mathbb{E}\left(\mathrm{tr}\{[x_t - \hat{x}_t(\mathcal{S})]^{\mathsf{T}} \Theta_t [x_t - \hat{x}_t(\mathcal{S})]\}\right) \\ &= \sum_{t=1}^{T} \mathbb{E}\left(\|K_t x_t - K_t \hat{x}_t(\mathcal{S})\|^2_{M_t}\right) \end{aligned} \quad (14)$$

since $\Sigma_{t|t}(\mathcal{S}) = \mathbb{E}[(x_t - \hat{x}_t(\mathcal{S}))(x_t - \hat{x}_t(\mathcal{S}))^{\mathsf{T}}]$ and $\Theta_t = K_t^{\mathsf{T}} M_t K_t$. From (14), each $\mathrm{tr}[\Theta_t \Sigma_{t|t}(\mathcal{S})]$ captures the mismatch between the imperfect state-information controller $u_t(\mathcal{S}) = K_t \hat{x}_t(\mathcal{S})$ (which is only aware of the measurements from the active sensors) and the perfect state-information controller $K_t x_t$. That is, while standard sensor selection minimizes the estimation covariance, for instance by minimizing

$$\sum_{t=1}^{T} \mathrm{tr}[\Sigma_{t|t}(\mathcal{S})] \triangleq \sum_{t=1}^{T} \mathbb{E}\left(\|x_t - \hat{x}_t(\mathcal{S})\|_2^2\right) \quad (15)$$

the proposed LQG cost formulation selectively minimizes the estimation error focusing on the states that are most informative for control purposes. For example, the mismatch contribution in (14) of any $x_t - \hat{x}_t(\mathcal{S})$ in the null space of K_t is zero; accordingly, the proposed sensor design approach has no incentive in activating sensors to observe states, which are irrelevant for control purposes.

B. Inapproximability of Optimal Sensing Design

Theorem 2 (Inapproximability): If *NP*≠*P*, then there is no polynomial time algorithm for Problems 1 and 2 that returns an approximate solution within a constant factor from the optimal. This remains true, even if all sensors have cost 1.

We prove the theorem by reducing the inapproximable problem in [65]—sensor selection with cost constraints for optimal steady-state Kalman filtering error—to (10).

Motivated by the inapproximability of Problems 1 and 2, we next present practical algorithms, which in Section IV, we prove to enjoy per-instance suboptimality bounds.

C. Codesign Algorithms for Problem 1

We present a practical algorithm for the sensing-constrained LQG control Problem 1 (see Algorithm 1). The algorithm follows Theorem 1: it first computes a sensing design, and then a control design, as described below.

1) Sensing Design for Problem 1: Theorem 1 implies an optimal sensor design for Problem 1 can be computed by solving (10). To this end, Algorithm 1 first computes $\Theta_1, \Theta_2, \ldots, \Theta_T$ (see Algorithm 1's line 1). Next, since (10) is inapproximable (see Theorem 2),

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Algorithm 2: Sensing Design for Problem 1.

Input: Horizon T; system in (1); covariance $\Sigma_{1|1}$; LQG cost matrices Q_t and R_t in (5); sensors in (2); sensor budget b; sensor cost $c(i)$, for all $i \in \mathcal{V}$.

Output: Sensor set $\widehat{\mathcal{S}}$.

1: $\widehat{\mathcal{S}}_1 \leftarrow \arg\min_{i\in\mathcal{V}, c(i)\leq b} \sum_{t=1}^{T} \mathrm{tr}[\Theta_t \Sigma_{t|t}(\{i\})]$;
2: $\widehat{\mathcal{S}}_2 \leftarrow \emptyset$; $\mathcal{V}' \leftarrow \mathcal{V}$;
3: **while** $\mathcal{V}' \neq \emptyset$ and $c(\widehat{\mathcal{S}}_2) \leq b$ **do**
4: **for all** $a \in \mathcal{V}'$ **do**
5: $\widehat{\mathcal{S}}_{2,\alpha} \leftarrow \widehat{\mathcal{S}}_2 \cup \{a\}$; $\Sigma_{1|1}(\widehat{\mathcal{S}}_{2,\alpha}) \leftarrow \Sigma_{1|1}$;
6: **for all** $t = 1, \ldots, T$ **do**
7: $\Sigma_{t+1|t}(\widehat{\mathcal{S}}_{2,\alpha}) \leftarrow A_t \Sigma_{t|t}(\widehat{\mathcal{S}}_{2,\alpha}) A_t^{\mathsf{T}} + W_t$;
8: $\Sigma_{t|t}(\widehat{\mathcal{S}}_{2,\alpha}) \leftarrow$
9: $[\Sigma_{t|t-1}(\widehat{\mathcal{S}}_{2,\alpha})^{-1} + C_t(\widehat{\mathcal{S}}_{2,\alpha})^{\mathsf{T}} V_t(\widehat{\mathcal{S}}_{2,\alpha})^{-1} C_t(\widehat{\mathcal{S}}_{2,\alpha})]^{-1}$;
10: **end for**
11: $\mathrm{gain}_a \leftarrow \sum_{t=1}^{T} \mathrm{tr}\{\Theta_t[\Sigma_{t|t}(\widehat{\mathcal{S}}_2) - \Sigma_{t|t}(\widehat{\mathcal{S}}_{2,\alpha})]\}$;
12: **end for**
13: $s \leftarrow \arg\max_{a\in\mathcal{V}'} [\mathrm{gain}_a / c(a)]$;
14: $\widehat{\mathcal{S}}_2 \leftarrow \widehat{\mathcal{S}}_2 \cup \{s\}$;
15: $\mathcal{V}' \leftarrow \mathcal{V}' \setminus \{s\}$;
16: **end while**
17: **if** $c(\widehat{\mathcal{S}}_2) > b$ **then**
18: $\widehat{\mathcal{S}}_2 \leftarrow \widehat{\mathcal{S}}_2 \setminus \{s\}$;
19: **end if**
20: $\widehat{\mathcal{S}} \leftarrow \arg\min_{\mathcal{S}\in\{\widehat{\mathcal{S}}_1, \widehat{\mathcal{S}}_2\}} \sum_{t=1}^{T} \mathrm{tr}[\Theta_t \Sigma_{t|t}(\mathcal{S})]$.

Algorithm 1 calls a greedy algorithm (see Algorithm 2) to compute a solution to (10) (see Algorithm 1's line 2).

Algorithm 2 computes a solution to (10) as follows: first, Algorithm 2 creates two candidate active sensor sets $\widehat{\mathcal{S}}_1$ and $\widehat{\mathcal{S}}_2$ (see lines 1–2), of which only one will be selected as the solution to (10) (see line 20). In more detail, Algorithm 2's line 1 lets $\widehat{\mathcal{S}}_1$ be composed of a single sensor, namely the sensor $i \in \mathcal{V}$ that achieves the smallest value of the objective function in (10) and has a smaller cost than the budget b ($c(i) \leq b$). Then, Algorithm 2's line 2 initializes $\widehat{\mathcal{S}}_2$ with the empty set, and after the construction of $\widehat{\mathcal{S}}_2$ in Algorithm 2's lines 3–19, Algorithm 2's line 20 computes which of $\widehat{\mathcal{S}}_1$ and $\widehat{\mathcal{S}}_2$ achieves the smallest value for the objective function in (10), and returns this set as the solution to (10).

Specifically, Algorithm 2's lines 3–19 construct $\widehat{\mathcal{S}}_2$ as follows: at each iteration of the "while loop" (see lines 3–16) a sensor is greedily added to $\widehat{\mathcal{S}}_2$, as long as $\widehat{\mathcal{S}}_2$'s cost does not exceed b. Particularly, for each remaining sensor a in $\mathcal{V} \setminus \widehat{\mathcal{S}}_2$, the "for loop" (see lines 4–12) computes first the estimation covariance resulting by adding a in $\widehat{\mathcal{S}}_2$, and then the marginal gain in the objective function in (10) (see line 11). Afterwards, the sensor inducing the largest marginal gain (normalized by the sensor's cost) is selected (see line 13), and is added in $\widehat{\mathcal{S}}_2$ (see line 14). Finally, the "if" in lines 17–19 ensure $\widehat{\mathcal{S}}_2$ has cost at most b, by removing last sensor added in $\widehat{\mathcal{S}}_2$ if necessary.

2) Control Design for Problem 1: Theorem 1 implies that given a sensor set, the controls for Problem 1 can be computed according to (11). To this end, Algorithm 1 first computes $K_1, K_2, \ldots, K_T$ (see line 3), and then, at each time $t = 1, 2, \ldots, T$, the Kalman estimate of the current state x_t (see line 4), and the corresponding control (see line 5).

D. Codesign Algorithms for Problem 2

This section presents a practical algorithm for Problem 2 (see Algorithm 3). Since the algorithm shares steps with Algorithm 1, we focus on the different ones.

Algorithm 3: Joint Sensing and Control Design for Problem 2.

Input: Horizon T; system in (1); covariance $\Sigma_{1|1}$; LQG cost matrices Q_t and R_t in (5); LQG cost bound κl sensors in (2); sensor cost $c(i)$, for all $i \in \mathcal{V}$.

Output: Active sensors $\widehat{\mathcal{S}}$, and controls $\hat{u}_1, \hat{u}_2, \ldots, \hat{u}_T$.

1: Compute $\Theta_1, \Theta_2, \ldots, \Theta_T$ using (9).
2: Return $\widehat{\mathcal{S}}$ returned by Algorithm 4, which finds a solution to the optimization problem in (7) ;
3: Compute $K_1, K_2, \ldots, K_T$ using (9).
4: At each $t = 1 \ldots, T$, compute the Kalman estimate of x_t:

$$\hat{x}_t \triangleq \mathbb{E}[x_t | y_1(\widehat{\mathcal{S}}), y_2(\widehat{\mathcal{S}}), \ldots, y_t(\widehat{\mathcal{S}})];$$

5: At each $t = 1, \ldots, T$, return $\hat{u}_t = K_t \hat{x}_t$.

Particularly, as Algorithm 1 calls Algorithm 2 to solve (10), similarly, Algorithm 3 calls Algorithm 4 to solve (12). Algorithm 4 is similar to Algorithm 2, with the difference that Algorithm 4 selects sensors until the upper bound $\bar{\kappa}$ in (12) is met (Algorithm 4's line 3), whereas Algorithm 2 selects sensors up to the point the cost budget b is violated (Algorithm 2's line 3).

IV. Performance Guarantees for LQG Codesign

We now quantify the suboptimality and running time of Algorithms 1 and 3. Particularly, we prove that both algorithms enjoy per-instance suboptimality bounds,[6] and run in quadratic time. To this end, we present a notion of a supermodularity ratio (see Definition 3), which we use to prove the suboptimality bounds. We then establish connections between the ratio and control-theoretic quantities (see Theorem 5), and conclude that the algorithms' suboptimality bounds are nonvanishing under control-theoretic conditions encountered in most real-world systems (see Theorem 6).

A. Supermodularity Ratio

To present the definition of *the supermodularity ratio*, we start by defining *monotonicity* and *supermodularity*.

Definition 1 (Monotonicity [47]): Consider any finite set $\mathcal{V}$. The set function $f : 2^{\mathcal{V}} \mapsto \mathbb{R}$ is *nonincreasing* if and only if for any sets $\mathcal{A} \subseteq \mathcal{B} \subseteq \mathcal{V}$, it holds $f(\mathcal{A}) \geq f(\mathcal{B})$.

Definition 2 (Supermodularity [47, Proposition 2.1]): Consider any finite set $\mathcal{V}$. The set function $f : 2^{\mathcal{V}} \mapsto \mathbb{R}$ is *supermodular* if and only if for any sets $\mathcal{A} \subseteq \mathcal{B} \subseteq \mathcal{V}$, and any element $v \in \mathcal{V}$, it holds $f(\mathcal{A}) - f(\mathcal{A} \cup \{v\}) \geq f(\mathcal{B}) - f(\mathcal{B} \cup \{v\})$.

That is, f is supermodular if and only if it satisfies a diminishing returns property: for any $v \in \mathcal{V}$, the drop $f(\mathcal{A}) - f(\mathcal{A} \cup \{v\})$ diminishes as the set $\mathcal{A}$ grows.

Definition 3 (Supermodularity ratio [53]): Consider any finite set $\mathcal{V}$, and a nonincreasing set function $f : 2^{\mathcal{V}} \mapsto \mathbb{R}$. We define the *supermodularity ratio* of f as

$$\gamma_f \triangleq \min_{\mathcal{A}\subseteq\mathcal{B}\subseteq\mathcal{V}, v\in\mathcal{V}\setminus\mathcal{B}} \frac{f(\mathcal{A}) - f(\mathcal{A}\cup\{v\})}{f(\mathcal{B}) - f(\mathcal{B}\cup\{v\})}.$$

The supermodularity ratio γ_f measures how far f is from being supermodular. Particularly, γ_f takes values in [0,1], and if $\gamma_f = 1$, then $f(\mathcal{A}) - f(\mathcal{A} \cup \{v\}) \geq f(\mathcal{B}) - f(\mathcal{B} \cup \{v\})$, i.e., f is supermodular. Whereas, if $0 < \gamma_f < 1$, then $f(\mathcal{A}) - f(\mathcal{A} \cup \{v\}) \geq \gamma_f [f(\mathcal{B}) - f(\mathcal{B} \cup \{v\})]$, i.e., γ_f captures how much ones needs to discount $f(\mathcal{B}) - f(\mathcal{B} \cup \{v\})$, such that $f(\mathcal{A}) - f(\mathcal{A} \cup \{v\})$ is at least $f(\mathcal{B}) - f(\mathcal{B} \cup \{v\})$. In this case, f is called *approximately (or weakly) supermodular* [66].

[6]Instead of constant suboptimality bounds across *all* instances, which is impossible due to Theorem 2.

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Algorithm 4: Sensing Design for Problem 2.

Input: Horizon T; system in (1); covariance $\Sigma_{1|1}$; LQG cost matrices Q_t and R_t in (5); LQG cost bound κl sensors in (2); sensor cost $c(i)$, for all $i \in \mathcal{V}$.

Output: Active sensors $\widehat{\mathcal{S}}$.

$\bar{\kappa} \leftarrow \kappa - \text{tr}(\Sigma_{1|1}N_1) - \sum_{t=1}^{T} \text{tr}(W_t S_t)$
$\widehat{\mathcal{S}} \leftarrow \emptyset$; $\mathcal{V}' \leftarrow \mathcal{V}$;
while $\mathcal{V}' \neq \emptyset$ and $\sum_{t=1}^{T} \text{tr}[\Theta_t \Sigma_{t|t}(\widehat{\mathcal{S}})] > \bar{\kappa}$ **do**
  for all $a \in \mathcal{V}'$ **do**
    $\widehat{\mathcal{S}}_\alpha \leftarrow \widehat{\mathcal{S}} \cup \{a\}$; $\Sigma_{1|1}(\widehat{\mathcal{S}}_\alpha) \leftarrow \Sigma_{1|1}$;
    for all $t = 1, \ldots, T$ **do**
      $\Sigma_{t+1|t}(\widehat{\mathcal{S}}_\alpha) \leftarrow A_t \Sigma_{t|t}(\widehat{\mathcal{S}}_\alpha) A_t^\intercal + W_t$;
      $\Sigma_{t|t}(\widehat{\mathcal{S}}_\alpha) \leftarrow$
        $[\Sigma_{t|t-1}(\widehat{\mathcal{S}}_\alpha)^{-1} + C_t(\widehat{\mathcal{S}}_\alpha)^\intercal V_t(\widehat{\mathcal{S}}_\alpha)^{-1} C_t(\widehat{\mathcal{S}}_\alpha)]^{-1}$;
    end for
    $\text{gain}_a \leftarrow \sum_{t=1}^{T} \text{tr}\{\Theta_t[\Sigma_{t|t}(\widehat{\mathcal{S}}) - \Sigma_{t|t}(\widehat{\mathcal{S}}_\alpha)]\}$;
  end for
  $s \leftarrow \arg\max_{a \in \mathcal{V}'} [\text{gain}_a / c(a)]$;
  $\widehat{\mathcal{S}} \leftarrow \widehat{\mathcal{S}} \cup \{s\}$;
  $\mathcal{V}' \leftarrow \mathcal{V}' \setminus \{s\}$;
end while

B. Performance Analysis for Algorithm 1

We quantify Algorithm 1's running time and suboptimality, using the notion of the supermodularity ratio. We use the notation.

- $g(\mathcal{S})$ is the optimal value of $h[\mathcal{S}, u_{1:T}(\mathcal{S})]$ across all $u_{1:T}(\mathcal{S})$, given any $\mathcal{S}$:
$$g(\mathcal{S}) \triangleq \min_{u_{1:T}(\mathcal{S})} h[\mathcal{S}, u_{1:T}(\mathcal{S})]. \tag{16}$$
- $h^\star \triangleq \min_{\mathcal{S} \subseteq \mathcal{V}, u_{1:T}(\mathcal{S})} h[\mathcal{S}, u_{1:T}(\mathcal{S})]$, s.t. $c(\mathcal{S}) \leq b$, i.e., the optimal value of Problem 1.
- $b^\star \triangleq \min_{\mathcal{S} \subseteq \mathcal{V}, u_{1:T}(\mathcal{S})} c(\mathcal{S})$, s.t. $h[\mathcal{S}, u_{1:T}(\mathcal{S})] \leq \kappa$, i.e., the optimal value of Problem 2.

Theorem 3 (Performance of Algorithm 1): Algorithm 1 returns a sensor set $\widehat{\mathcal{S}}$ and control policies $u_{1:T}(\widehat{\mathcal{S}})$ such that

$$\frac{h[\emptyset, u_{1:T}(\emptyset)] - h[\widehat{\mathcal{S}}, u_{1:T}(\widehat{\mathcal{S}})]}{h[\emptyset, u_{1:T}(\emptyset)] - h^\star} \geq \max\left[\frac{\gamma_g}{2}\left(1 - e^{-\gamma_g}\right), 1 - e^{-\gamma_g c(\widehat{\mathcal{S}})/b}\right] \tag{17}$$

where γ_g is the supermodularity ratio of $g(\mathcal{S})$ in (16).

Moreover, Algorithm 1 runs in $O(|\mathcal{V}|^2 T n^{2.4})$ time.

In (17), $h[\emptyset, u_{1:T}(\emptyset)] - h[\widehat{\mathcal{S}}, u_{1:T}(\widehat{\mathcal{S}})]$ quantifies the gain from selecting $\widehat{\mathcal{S}}$, and in (17) the right-hand side guarantees that the gain is close to the optimal $h[\emptyset, u_{1:T}(\emptyset)] - h^\star$.[7] Specifically, when either of the bounds in the right-hand side of (17) is 1, then the algorithm returns an optimal solution.

For comparison, in Fig. 1, we plot the bounds for $c(\widehat{\mathcal{S}})/b \in \{2/5, 1, 2\}$ and all $\gamma_g \in [0, 1]$. We observe that $1 - e^{-\gamma_g c(\widehat{\mathcal{S}})/b}$ dominates $\gamma_g/2(1 - e^{-\gamma_g})$ for $c(\widehat{\mathcal{S}})/b > 2/5$. Moreover, as $c(\widehat{\mathcal{S}})/b$ and γ_g increase, then $1 - e^{-\gamma_g c(\widehat{\mathcal{S}})/b}$ tends to 1, in which case, Algorithm 1 returns an optimal solution.

Remark 4 (Novelty of algorithm and bounds): Algorithm 1 is the first scalable algorithm for Problem 1. Notably, although Algorithm 2 (used in Algorithm 1) is the same as [51, Algorithm 1], the latter was introduced for *exactly supermodular* optimization, instead of *approximately supermodular optimization*, which is the optimization framework in this article. Therefore, one of our contributions with Theorem 3 is to prove Algorithm 2 maintains suboptimality bounds even for approximately supermodular optimization. The novel bounds in Theorem 3 also improve upon the previously known [51], [62] for exactly supermodular optimization: particularly, our bounds can become $1 - 1/e$ for supermodular optimization (the closer $c(\widehat{\mathcal{S}})/b$ is to 1), tightening the known $1/2(1 - 1/e)$ [51], [62]. Noticeably, $1 - 1/e$ is the best possible bound in polynomial time for submodular optimization subject to *cardinality* constraints [47], instead of the general *cost* constraints in this article. That way, our analysis equates the approximation difficulty of *cost* and *cardinality* constrained optimization for the first time (among all algorithms with at most quadratic running time in the number of available elements in $\mathcal{V}$, i.e., those in [47], [51], [62], and ours).

All in all, Theorem 3 guarantees that Algorithm 1 achieves a close-to-optimal solution for Problem 1, whenever $\gamma_g > 0$. In Section IV-D, we present conditions such that $\gamma_g > 0$.

Finally, Theorem 3 also quantifies the scalability of Algorithm 1. Particularly, Algorithm 1's running time $O(|\mathcal{V}|^2 T n^{2.4})$ is in the worst case quadratic in the number of available sensors $\mathcal{V}$ (when all must be chosen active), and linear in the Kalman filter's running time: specifically, the multiplier $Tn^{2.4}$ is due to the complexity of computing all $\Sigma_{t|t}$ for $t = 1, 2, \ldots, T$ [1, Appendix E].

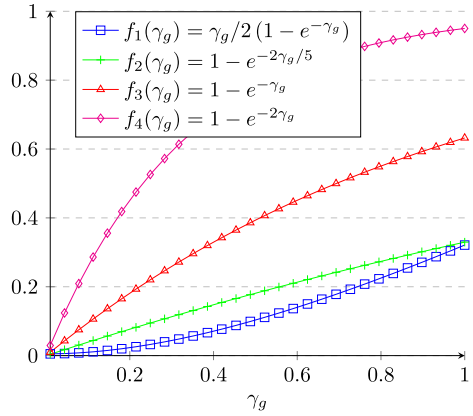


Fig. 1. Plot of $f_i(\gamma_g)$, where $i = 1, 2, 3, 4$, for increasing values of γ_g (each f_i is defined in the figure's legend). By Definition 3 of γ_g, γ_g takes values between 0 and 1.

C. Performance Analysis for Algorithm 3

Theorem 4 (Performance of Algorithm 3): Consider Algorithm 3 returns a sensor set $\widehat{\mathcal{S}}$ and control policies $u_{1:T}(\widehat{\mathcal{S}})$. Let s_l be the last sensor added to $\widehat{\mathcal{S}}$. Then,

$$h[\widehat{\mathcal{S}}, u_{1:T}(\widehat{\mathcal{S}})] \leq \kappa \tag{18}$$

$$c(\widehat{\mathcal{S}}) \leq c(s_l) + \frac{1}{\gamma_g} \log\left(\frac{h[\emptyset, u_{1:T}(\emptyset)] - \kappa}{h[\widehat{\mathcal{S}}_{l-1}, u_{1:T}(\widehat{\mathcal{S}}_{l-1})] - \kappa}\right) b^\star \tag{19}$$

where $\widehat{\mathcal{S}}_{l-1} \triangleq \widehat{\mathcal{S}} \setminus \{s_l\}$.

Additionally, Algorithm 3 runs in $O(|\mathcal{V}|^2 T n^{2.4})$ time.

Remark 5 (Novelty of algorithm and bound): Algorithm 3 is the first scalable algorithm for Problem 2. Importantly, Algorithm 4, used in Algorithm 3, is the first scalable algorithm with suboptimality guarantees for the problem of *the minimal cost set selection* where a bound to an *approximately* supermodular g must be met. Particularly, Algorithm 4, generalizes previous algorithms [48] that focus instead on *minimal cardinality* set selection subject to bounds on an *exactly*

[7] Even if no sensors are active, observe $h[\emptyset, u_{1:T}(\emptyset)]$ is well defined and finite, since it is the LQG cost over a *finite* horizon T.

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supermodular function g (in which case, $\gamma_g = 1$). Notably, for $\gamma_g = 1$, in (19)'s bound recovers the guarantee established in [48, Th. 1].

All in all, (18) implies Algorithm 3 returns a solution to Problem 2 with the prescribed LQG performance. And parallel to (17), (19) implies for $\gamma_g > 0$ that Algorithm 3 achieves a close-to-optimal sensor cost.

D. Conditions for $\gamma_g > 0$

We provide control-theoretic conditions such that γ_g is nonzero, in which case both Algorithm 1 and Algorithm 3 guarantee a close-to-optimal performance. Particularly, we first prove that if $\sum_{t=1}^{T} \Theta_t \succ 0$, then γ_g is nonzero. Afterwards, we show that the condition holds true in all problem instances one typically encounters in the real-world. Specifically, we prove $\sum_{t=1}^{T} \Theta_t \succ 0$ holds whenever zero control would result in a suboptimal behavior for the system; that is, we prove $\sum_{t=1}^{T} \Theta_t \succ 0$ holds in all systems where LQG control improves system performance.

Theorem 5 (Nonzero computable bound for the supermodularity ratio γ_g): For any sensor $i \in \mathcal{V}$, let $\bar{C}_{i,t} \triangleq V_{i,t}^{-1/2} C_{i,t}$ be the *whitened* measurement matrix. If the strict inequality $\sum_{t=1}^{T} \Theta_t \succ 0$ holds, then $\gamma_g \neq 0$. Additionally, if we assume $\mathrm{tr}(\bar{C}_{i,t}\bar{C}_{i,t}^{\mathsf{T}}) = 1$, and $\mathrm{tr}[\Sigma_{t|t}(\emptyset)] \leq \lambda_{\max}^2[\Sigma_{t|t}(\emptyset)]$, then

$$\gamma_g \geq \frac{\lambda_{\min}(\sum_{t=1}^{T} \Theta_t)}{\lambda_{\max}(\sum_{t=1}^{T} \Theta_t)} \frac{\min_{t\in\{1,2,\ldots,T\}} \lambda_{\min}^2[\Sigma_{t|t}(\mathcal{V})]}{\max_{t\in\{1,2,\ldots,T\}} \lambda_{\max}^2[\Sigma_{t|t}(\emptyset)]} \frac{1 + \min_{i\in\mathcal{V}, t\in\{1,2\ldots,T\}} \lambda_{\min}[\bar{C}_i \Sigma_{t|t}(\mathcal{V}) \bar{C}_i^{\mathsf{T}}]}{2 + \max_{i\in\mathcal{V}, t\in\{1,2\ldots,T\}} \lambda_{\max}[\bar{C}_i \Sigma_{t|t}(\emptyset) \bar{C}_i^{\mathsf{T}}]}. \tag{20}$$

Equation (20) suggests ways γ_g can increase, and, correspondingly, the bounds for Algorithm 1 and of Algorithm 3 can improve: when $\lambda_{\min}(\sum_{t=1}^{T} \Theta_t)/\lambda_{\max}(\sum_{t=1}^{T} \Theta_t)$ increases to 1, then the right-hand side in (20) increases. Therefore, since each Θ_t weight the states depending on their relevance for control purposes (see Remark 3), the right-hand side in (20) increases when all the directions in the state space become equally important for control purposes. Indeed, in the extreme case, where $\lambda_{\max}(\Theta_t) = \lambda_{\min}(\Theta_t) = \lambda$, the objective function in (10) becomes

$$\sum_{t=1}^{T} \mathrm{tr}[\Theta_t \Sigma_{t|t}(\mathcal{S})] = \lambda \sum_{t=1}^{T} \mathrm{tr}[\Sigma_{t|t}(\mathcal{S})]$$

which matches the cost function in the classical sensor selection where all states are equally important [per (15)].

Theorem 5 states γ_g is nonzero whenever $\sum_{t=1}^{T} \Theta_t \succ 0$. To provide insight on the type of control problems for which this result holds, next we translate the technical condition $\sum_{t=1}^{T} \Theta_t \succ 0$ into an equivalent control-theoretic condition.

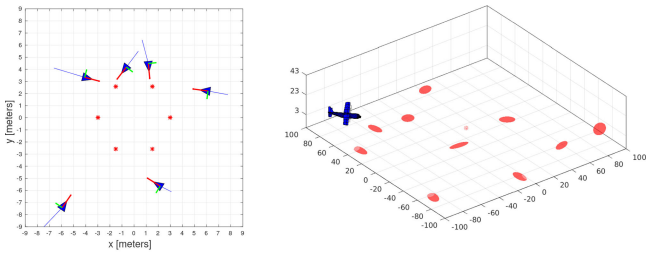
Theorem 6 (Control-theoretic condition for near-optimal codesign): Consider the (noiseless, perfect state-information) LQG problem where at any $t = 1, 2, \ldots, T$, the state x_t is known to each controller u_t and the process noise w_t is zero, i.e., the optimal control problem

$$\min_{u_{1:T}} \sum_{t=1}^{T} \left[\|x_{t+1}\|_{Q_t}^2 + \|u_t(x_t)\|_{R_t}^2\right]\Big|_{\Sigma_{t|t}=W_t=0}. \tag{21}$$

Let A_t be invertible for all $t = 1, 2, \ldots, T$; the strict inequality $\sum_{t=1}^{T} \Theta_t \succ 0$ holds if and only if for all nonzero initial conditions x_1, the all-zeroes control policy $u_{1:T}^{\circ} \triangleq (0, 0, \ldots, 0)$ is not an optimal solution to (21)

$$u_{1:T}^{\circ} \notin \arg\min_{u_{1:T}} \sum_{t=1}^{T} \left[\|x_{t+1}\|_{Q_t}^2 + \|u_t(x_t)\|_{R_t}^2\right]\Big|_{\Sigma_{t|t}=W_t=0}.$$

Theorem 6 suggests $\sum_{t=1}^{T} \Theta_t \succ 0$ holds if and only if for any nonzero initial condition x_1 the all-zeroes control policy $u_{1:T}^{\circ} = (0, 0, \ldots, 0)$ is suboptimal for the noiseless, perfect state-information LQG problem. Intuitively, this encompasses most practical control design problems where a zero controller would result in a suboptimal behavior of the system (LQG control design itself would be unnecessary in the case where a zero controller, i.e., no control action, can already attain the desired system performance).



(a) (b)

Fig. 2. Applications of the LQG control and sensing co-design framework. (a) Formation control. (b) Robot navigation.

Overall, Algorithm 1 and Algorithm 3 are the first scalable algorithms for Problem 1 and Problem 2, respectively, and they achieve nonvanishing per-instance performance guarantees.

V. Numerical Evaluations

We consider two applications for the LQG control and sensing codesign framework: *formation control* and *autonomous navigation*. We present a Monte Carlo analysis for both, which demonstrates: (i) the proposed sensor selection strategy is near-optimal; particularly, the resulting LQG cost matches the optimal selection in all instances for which the optimal could be computed via a brute-force approach; (ii) a more naive selection, which attempts to minimize the state estimation covariance [24] (rather than the LQG cost) has degraded LQG performance, often comparable to a random selection; (iii) in the considered instances, a clever selection of a small subset of sensors can ensure an LQG cost that is close to the one obtained by using all available sensors, hence, providing an effective alternative for control under sensing constraints.

A. Sensing-Constrained Formation Control

1) Simulation Setup: The application scenario is illustrated in Fig. 2(a). A team of n agents (blue triangles) moves in 2D. At $t = 1$, the agents are randomly deployed in a $10\,\mathrm{m} \times 10\,\mathrm{m}$ square. Their objective is to reach a target formation shape (red stars); in Fig. 2(a) the desired formation has an hexagonal shape, while in general for a formation of n, the desired formation is an equilateral polygon with n vertices. Each robot is modeled as a double-integrator, with state $x_i = [p_i\ v_i]^{\mathsf{T}} \in \mathbb{R}^4$ (p_i is agent is position, and v_i its velocity), and can control its acceleration $u_i \in \mathbb{R}^2$. The process noise is a diagonal matrix $W = \mathrm{diag}([1e^{-2},\ 1e^{-2},\ 1e^{-4},\ 1e^{-4}])$. Each robot i is equipped with a GPS, which measures the agent position p_i with a covariance $V_{\mathrm{gps},i} = 2 \cdot \mathbf{I}_2$. Moreover, the agents are equipped with lidars allowing each agent i to measure the relative position of another agent j with covariance $V_{\mathrm{lidar},ij} = 0.1 \cdot \mathbf{I}_2$. The agents have limited on-board resources; hence, they want to activate only k sensors.

For our tests, we consider two setups. In the first, named *homogeneous formation control*, the LQG weight matrix Q is a block diagonal matrix with 4×4 blocks, and each block i chosen as $Q_i = 0.1 \cdot \mathbf{I}_4$; since each block of Q weights equally the tracking error of a robot, in the homogeneous case the tracking error of all agents is equally important. In the second setup, named *heterogeneous formation control*, Q is chose as above, except for one of the agents, say robot 1, for which we choose $Q_1 = 10 \cdot \mathbf{I}_4$; this setup models the case in which each agent has a different role or importance; hence, one weights differently the tracking error of the agents. In both cases, the matrix R is chosen to be the identity matrix. The simulation is carried on over T time steps, and T is also chosen as LQG horizon. Results are averaged over 100 Monte

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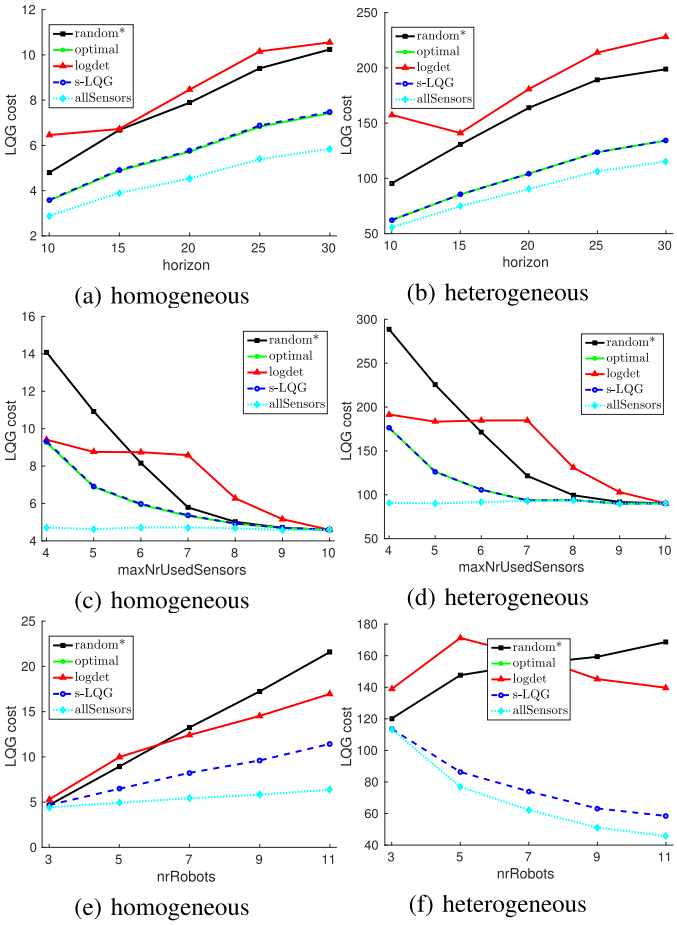


Fig. 3. LQG cost for increasing (a)-(b) control horizon T, (c)-(d) number of used sensors k (all sensors are considered to have sensor-cost 1), and (e)-(f) number of agents n. Statistics are reported for the homogeneous formation control setup (left column) and the heterogeneous setup (right column).

Carlo runs: at each run we randomize the initial estimation covariance $\Sigma_{1|1}$.

2) Compared Techniques: We compare five techniques. All techniques use an LQG-based estimator and controller, and they only differ by the selections of the active sensors. The first approach is the optimal sensor selection, denoted as `optimal`, which attains the minimum in (10), and which we compute by enumerating all possible subsets. The second approach is a pseudorandom sensor selection, denoted as `random`*, which selects all the GPS measurements and a random subset of the lidar measurements. The third approach, denoted as `logdet`, selects sensors so to minimize the average $\log\det$ of the estimation covariance over the horizon; this approach resembles [24] and is agnostic to the control task. The fourth approach is the proposed sensor selection strategy (see Algorithm 2) and is denoted as `s-LQG`. Finally, we also report the LQG performance when all sensors are selected. This approach is denoted as `allSensors`.

3) Results: The results of the numerical analysis are reported in Fig. 3. When not specified otherwise, we consider a formation of $n = 4$ agents, which can only use a total of $k = 6$ sensors, and a control horizon $T = 20$. Fig. 3(a) shows the LQG cost for the homogeneous case and for increasing horizon. We note that, in all tested instance, the proposed approach `s-LQG` matches the optimal selection `optimal`, and both approaches are relatively close to `allSensors`, which selects all the available sensors. On the other hand, `logdet` leads to worse tracking performance, and is often close to `random`*. These considerations are confirmed by the heterogeneous setup, in Fig. 3(b). In this case, the separation between our proposed approach and `logdet` becomes even larger; the intuition is that the heterogeneous case rewards differently the tracking errors at different agents; hence, while `logdet` attempts to equally reduce the estimation error across the formation, the proposed approach `s-LQG` selects sensors in a task-oriented fashion, since the matrices Θ_t for all $t = 1, 2, \ldots, T$ in the cost function in (10) incorporate the LQG weight matrices.

Fig. 3(c) shows the LQG cost attained for increasing number of selected sensors k and for the homogeneous case. For increasing number of sensors all techniques converge to `allSensors` (since the entire ground set is selected). Fig. 3(d) shows the same statistics for the heterogeneous case. Now, `s-LQG` matches `allSensors` earlier, starting at $k = 7$; intuitively, in the heterogeneous case, adding more sensors may have marginal impact on the LQG cost (e.g., if the cost rewards a small tracking error for robot 1, it may be of a little value to take a lidar measurement between robots 3 and 4). This further stresses the importance of the proposed framework as a parsimonious way to control a system with minimal resources.

Fig. 3(e) and (f) shows the LQG cost attained by the compared techniques for increasing number of agents. `optimal` quickly becomes intractable to compute; hence, we omit values beyond $n = 4$. In both figures, the separation among the techniques increases with the number of agents, since the set of available sensors quickly increases with n. In the heterogeneous case `s-LQG` remains relatively close to `allSensors`, implying that for the purpose of LQG control, using a cleverly selected small subset of sensors still ensures excellent tracking performance.

B. Resource-Constrained Robot Navigation

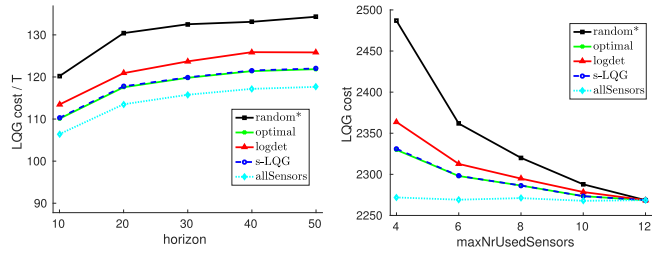
1) Simulation Setup: The second application scenario is illustrated in Fig. 2(b). An unmanned aerial robot (UAV) moves in a 3-D space, starting from a randomly selected location. The objective of the UAV is to land, and specifically, to reach $[0,\ 0,\ 0]$ with zero velocity. The UAV is modeled as a double-integrator, with state $x = [p\ v]^\mathsf{T} \in \mathbb{R}^6$ (p is the position, while v its velocity), and can control its acceleration $u \in \mathbb{R}^3$. The process noise is $W = \mathbf{I}_6$. The UAV is equipped with multiple sensors. It has an on-board GPS, measuring the UAV position p with a covariance $2 \cdot \mathbf{I}_3$, and an altimeter, measuring only the last component of p (altitude) with standard deviation $0.5\,\mathrm{m}$. Moreover, the UAV can use a stereo camera to measure the relative position of ℓ landmarks on the ground; we assume the location of each landmark to be known approximately and we associate to each landmark an uncertainty covariance [red ellipsoids in Fig. 2(b)], which is randomly generated at the beginning of each run. The UAV has limited on-board resources; hence, it wants to use only a few of sensing modalities. For instance, the resource-constraints may be due to the power consumption of the GPS and the altimeter, or may be due to computational constraints that prevent to run multiple object-detection algorithms to detect all landmarks on the ground. We consider two sensing-constrained scenarios: (i) all sensors to have the same cost (equal to 1), in which case, the UAV can activate at most k sensors; (ii) the sensors to have heterogeneous costs: particularly, the GPS's cost is set equal to 3; the altimeter's cost is set equal to 2; and each landmark's cost is set equal to 1.

We use $Q = \mathrm{diag}([1e^{-3},\ 1e^{-3},\ 10,\ 1e^{-3},\ 1e^{-3},\ 10])$ and $R = \mathbf{I}_3$. The structure of Q reflects the fact that during landing we are particularly interested in controlling the vertical direction and the vertical velocity (entries with larger weight in Q), while we are less interested in controlling accurately the horizontal position and velocity (assuming a sufficiently large landing site). In the following, we present results averaged over 100 Monte Carlo runs: in each run, we randomize the covariances describing the landmark position uncertainty.

2) Compared Techniques: We consider the five techniques discussed in the previous section.

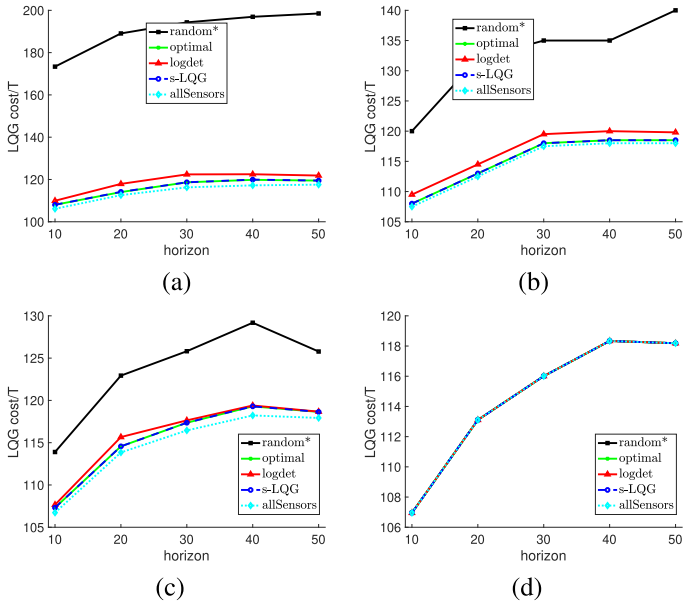
3) Results: The results of our numerical analysis are reported in Fig. 4 for the case where all sensors have the same sensor-cost, and in Fig. 5 for the case where sensors have different costs. When not

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(a) heterogeneous (b) heterogeneous

Fig. 4. LQG cost for increasing (a) horizon T, and (b) number of used sensors k (all sensors are considered to have cost 1).



(a) (b) (c) (d)

Fig. 5. LQG cost for increasing horizon T and for various sensing budgets b. (a) Budget $b = 6$. (b) Budget $b = 8$. (c) Budget $b = 10$. (d) Budget $b = 15$.

specified otherwise, we consider a total of $k = 3$ sensors to be selected, and a control horizon $T = 20$. In Fig. 4(a), we plot the LQG cost normalized by the horizon, which makes more visible the differences among the techniques. Similarly to the formation control example, `s-LQG` matches the optimal selection `optimal`, while `logdet` and `random`* have suboptimal performance. Fig. 4(b) shows the LQG cost attained by the compared techniques for increasing number of selected sensors k. All techniques converge to `allSensors` for increasing k, but in the regime in which few sensors are used `s-LQG` still outperforms alternative sensor selection schemes, and matches `optimal`.

Fig. 5 shows the LQG cost attained by the compared techniques for increasing control horizon and various sensor cost budgets b. Similarly to Fig. 4, `s-LQG` has the same performance as `optimal`, whereas `logdet` and `random`* have suboptimal performance. Notably, for $b = 15$ all sensors can be chosen; for this reason in Fig. 5(d) all compared techniques (but the random) have the same performance.

VI. Concluding Remarks

We addressed an LQG control and sensing codesign problem, where one jointly designs control and sensing policies under resource constraints. The problem is central in modern IoT and IoBT control applications, ranging from large-scale networked systems to miniaturized robotic networks. Motivated by the inapproximability of the problem, we provided the first scalable algorithms with per-instance suboptimality bounds. Importantly, the bounds are nonvanishing under general control-theoretic conditions, encountered in most real-world systems. To this end, we also extended the literature on supermodular optimization: by providing scalable algorithms for optimizing approximately supermodular functions subject to heterogeneous cost constraints; and by providing novel suboptimality bounds that improve the known bounds even for exactly supermodular optimization.

The article opens several avenues for future research. First, the development of distributed implementations of the proposed algorithms would offer computational speedups. Second, other codesign problems are interesting to be explored, such as the codesign of control-sensing-actuation. Third, while we provide bounds on an approximate sensor design against optimal design, one could provide bounds against the case where all sensors are used [67]. Finally, in adversarial or failure-prone scenarios, one must account for sensor failures; to this end, one could leverage recent results on *robust combinatorial optimization* [68].

Appendix A
Preliminary Facts

Lemma 1 ([69, Proposition 8.5.5]): Consider two positive definite matrices A_1 and A_2. If $A_1 \preceq A_2$, then $A_2^{-1} \preceq A_1^{-1}$.

Lemma 2 (Trace inequality [69, Proposition 8.4.13]): Consider a symmetric A, and a positive semidefinite B. Then,

$$\lambda_{\min}(A)\mathrm{tr}(B) \leq \mathrm{tr}(AB) \leq \lambda_{\max}(A)\mathrm{tr}(B).$$

Lemma 3 (Woodbury identity [69, Corollary 2.8.8]): Consider A, C, U, and V such that A, C, and $A + UCV$ are invertible. Then,

$$(A + UCV)^{-1} = A^{-1} - A^{-1}U(C^{-1} + VA^{-1}U)^{-1}VA^{-1}.$$

Lemma 4 ([69, Proposition 8.5.12]): Consider two symmetric matrices A_1 and A_2, and a positive semidefinite matrix B. If $A_1 \preceq A_2$, then $\mathrm{tr}(A_1B) \leq \mathrm{tr}(A_2B)$.

Lemma 5 ([1, Appendix E]): For any sensors $\mathcal{S}$, $\Sigma_{t|t}(\mathcal{S})$ is the solution of the Kalman filtering recursion

$$\begin{aligned} \Sigma_{t|t}(\mathcal{S}) &= [\Sigma_{t|t-1}(\mathcal{S})^{-1} + C_t(\mathcal{S})^{\mathsf{T}}V_t(\mathcal{S})^{-1}C_t(\mathcal{S})]^{-1} \\ \Sigma_{t+1|t}(\mathcal{S}) &= A_t\Sigma_{t|t}(\mathcal{S})A_t^{\mathsf{T}} + W_t \end{aligned} \tag{22}$$

with boundary condition $\Sigma_{1|1}(\mathcal{S}) = \Sigma_{1|1}$.

Lemma 6 ([63, Lemma 6]): Consider two sensor sets $\mathcal{S}_1, \mathcal{S}_2 \subseteq \mathcal{V}$. If $\mathcal{S}_1 \subseteq \mathcal{S}_2$, then $\Sigma_{1|1}(\mathcal{S}_1) \succeq \Sigma_{1|1}(\mathcal{S}_2)$.

Lemma 7 ([63, Corollary 1]): Let $\Sigma_{t|t}$ be defined as in (22) with boundary condition $\Sigma_{1|1}$; similarly, let $\bar{\Sigma}_{t|t}$ be defined as in (22) with boundary condition $\bar{\Sigma}_{1|1}$. If $\Sigma_{t|t} \preceq \bar{\Sigma}_{t|t}$, then $\Sigma_{t+i|t+i} \preceq \bar{\Sigma}_{t+i|t+i}$ for any positive integer i.

Lemma 8 ([63, Corollary 2]): Let $\Sigma_{t|t}$ be defined as in (22) with boundary condition $\Sigma_{1|1}$; similarly, let $\bar{\Sigma}_{t|t}$ be defined as in (22) with boundary condition $\bar{\Sigma}_{1|1}$. If $\Sigma_{t|t} \preceq \bar{\Sigma}_{t|t}$, then $\Sigma_{t+i|t+i-1} \preceq \bar{\Sigma}_{t+i|t+i-1}$ for any positive integer i.

Lemma 9: Consider positive real numbers a, b, γ, $a_1, a_2, \ldots, a_n$ such that $\sum_{i=1}^n a_i = a$. Then,

$$f(a_1, a_2, \ldots, a_n) = 1 - \prod_{i=1}^{n} \left(1 - \gamma \frac{a_i}{b}\right)$$

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has its minimum at $a_1 = a_2 = \ldots = a_n = a/n$, and

$$f(a/n, a/n, \ldots, a/n) = 1 - \left(1 - \frac{a\gamma}{bn}\right)^n \geq 1 - e^{-a\gamma/b}.$$

Proof of Lemma 9: The proof is obtained using the method of Lagrange multipliers, and is omitted (for a complete proof, see [64, Proof of Lemma 9]) ■

Lemma 10 (Monotonicity of cost function in (10) [63, Proposition 2]): Consider $\sum_{t=1}^{T} \text{tr}(\Theta_t \Sigma_{t|t}(\mathcal{S}))$ in (10). If $\mathcal{S}_1 \subseteq \mathcal{S}_2$, then $\sum_{t=1}^{T} \text{tr}(\Theta_t \Sigma_{t|t}(\mathcal{S}_1)) \geq \sum_{t=1}^{T} \text{tr}(\Theta_t \Sigma_{t|t}(\mathcal{S}_2))$.

APPENDIX B
PROOF OF THEOREM 1

A. Proof of Part (1) of Theorem 1

Lemma 11: Consider any $\mathcal{S}$, and let $u^{\star}_{1:T}(\mathcal{S})$ be the vector of control policies $(K_1 \hat{x}_1(\mathcal{S}), K_2 \hat{x}_2(\mathcal{S}), \ldots, K_T \hat{x}_T(\mathcal{S}))$. Then, $u^{\star}_{1:T}(\mathcal{S})$ is an optimal control policy:

$$u^{\star}_{1:T}(\mathcal{S}) \in \arg\min_{u_{1:T}(\mathcal{S})} h[\mathcal{S}, u_{1:T}(\mathcal{S})] \tag{23}$$

and, particularly, $u^{\star}_{1:T}(\mathcal{S})$ attains an LQG cost equal to

$$h[\mathcal{S}, u^{\star}_{1:T}(\mathcal{S})] = \mathbb{E}(\|x_1\|_{N_1}) + \sum_{t=1}^{T} \left\{ \text{tr}[\Theta_t \Sigma_{t|t}(\mathcal{S})] + \text{tr}\left(W_t S_t\right) \right\}. \tag{24}$$

Proof of Lemma 11: The proof follows Lemma 1's proof in [20], and can also be found in [64, Appendix B]. ■

Proof of Part (1) of Theorem 1: Equation (10) is a direct consequence of (24), since the value of Problem 1 is equal to $\min_{\mathcal{S} \subseteq \mathcal{V}, c(\mathcal{S}) \leq b} h[\mathcal{S}, u^{\star}_{1:T}(\mathcal{S})]$, and both $\mathbb{E}(\|x_1\|_{N_1}) = \text{tr}(\Sigma_{1|1} N_1)$ and $\sum_{t=1}^{T} \text{tr}(W_t S_t)$ are independent of $\mathcal{S}$. Finally, (11) directly follows from (23). ■

B. Proof of Part (2) of Theorem 1

Lemma 12: $\mathcal{S}^{\star}$, and $u^{\star}_{1:T}$ are a solution to Problem 2 if and only if they are a solution to

$$\min_{\mathcal{S} \subseteq \mathcal{V}, u_{1:T}(\mathcal{S})} c(\mathcal{S}), \quad \text{s.t.} \quad \min_{u_{1:T}(\mathcal{S})} h\left[\mathcal{S}, u_{1:T}(\mathcal{S})\right] \leq \kappa. \tag{25}$$

Proof of Lemma 12: We prove the lemma by contradiction. Particularly, let $\mathcal{S}^{\star}$ and $u^{\star}_{1:T}$ be a solution to Problem 2, and assume by contradiction that they are not to (25), which instead has solution $\widehat{\mathcal{S}}$ and $\widehat{u}_{1:T}$. By optimality of $\widehat{\mathcal{S}}$ and $\widehat{u}_{1:T}$ (and suboptimality of $\mathcal{S}^{\star}$ and $u^{\star}_{1:T}$) in (25), it follows $c(\widehat{\mathcal{S}}) < c(\mathcal{S}^{\star})$. In addition, $g(\widehat{\mathcal{S}}) \leq \kappa$, since $(\widehat{\mathcal{S}}, \widehat{u}_{1:T})$ must be feasible for (25). However, the latter implies $h(\widehat{\mathcal{S}}, \widehat{u}_{1:T}) \leq \kappa$. Therefore, $(\widehat{\mathcal{S}}, \widehat{u}_{1:T})$ is feasible for Problem 2 and has a better objective value with respect to the optimal solution $(\mathcal{S}^{\star}, u^{\star}_{1:T})$ (we already observed $c(\widehat{\mathcal{S}}) < c(\mathcal{S}^{\star})$), leading to contradiction.

For the other direction, now let $\mathcal{S}^{\star}$ and $u^{\star}_{1:T}$ be a solution to (25), and assume that they are not to Problem 2, which instead has solution $(\widehat{\mathcal{S}}, \widehat{u}_{1:T})$. By optimality of $(\widehat{\mathcal{S}}, \widehat{u}_{1:T})$ (and suboptimality of $\mathcal{S}^{\star}$ and $u^{\star}_{1:T}$) in Problem 2, it follows $c(\widehat{\mathcal{S}}) < c(\mathcal{S}^{\star})$. In addition, $h(\widehat{\mathcal{S}}, \widehat{u}_{1:T}) \leq \kappa$, since $(\widehat{\mathcal{S}}, \widehat{u}_{1:T})$ must be feasible for Problem 2, and, as a result, $g(\widehat{\mathcal{S}}) \leq \kappa$. Therefore, $(\widehat{\mathcal{S}}, \widehat{u}_{1:T})$ is feasible for (25) and has a better objective value with respect to the optimal solution $(\mathcal{S}^{\star}, u^{\star}_{1:T})$ (we already observed $c(\widehat{\mathcal{S}}) < c(\mathcal{S}^{\star})$), leading to contradiction. ■

Proof of Part (2) of Theorem 1: The proof follows from Lemma 11 and Lemma 12. ■

APPENDIX C
PROOF OF THEOREM 2

Consider a problem instance for Problem 1 and Problem 2, where $T = 1$, and $A_1 = B_1 = C_1 = Q_1 = R_1 = I$. Then, $\Theta_1 = \mathbf{I}/2$, and, as a result, the objective function in (10) becomes $1/2\text{tr}[\Sigma_{1|1}(\mathcal{S})]$. Now, choosing $\Sigma_{1|1}(\mathcal{S})$ to be the steady-state Kalman filtering matrix defined in [65, Th. 2], as well as, $c(\mathcal{S})$, b be as in [65, Th. 2], makes (10) and the optimization problem in [65] equivalent. But, the latter is inapproximable in polynomial time [65, Th. 2] (namely, unless NP=*P*, there is no polynomial time algorithm that guarantees a constant suboptimality bound). Therefore, (10) is too, and due to Theorem 1 both Problem 1 and Problem 2 as well.

APPENDIX D
PROOF OF THEOREM 3

For any $\mathcal{S}$, let $f(\mathcal{S}) \triangleq \sum_{t=1}^{T} \text{tr}[\Theta_t \Sigma_{t|t}(\mathcal{S})]$ be the objective function in (10), $\mathcal{S}^{\star}$ be a solution in (10), and $b^{\star} \triangleq c(\mathcal{S}^{\star})$. Let $\widehat{\mathcal{S}}_2$ be the set Algorithm 2 constructs by the end of line 19; let $\mathcal{G} \triangleq \widehat{\mathcal{S}}_2$. Let s_i be the ith element added in $\mathcal{G}$ during the ith iteration of Algorithm 2's "while loop" (see lines 3–16). Let $\mathcal{G}_i \triangleq \{s_1, s_2, \ldots, s_i\}$. Finally, consider Algorithm 2's "while loop" terminates after $l + 1$ iterations.

Algorithm 2's "while loop" terminates: (i) when $\mathcal{V}' = \emptyset$, that is, when all available sensors in $\mathcal{V}$ can been chosen by Algorithm 2 as active while satisfying the budget constraint b; and (ii) when $c(\mathcal{G}_{l+1}) > b$, that is, when the addition of s_{l+1} in $\mathcal{G}_l$ makes the cost of $\mathcal{G}_{l+1}$ to violate the budget b. Henceforth, we focus on the second scenario, which implies s_{l+1} will be removed by the "if" statement in Algorithm 2's lines 17–19 and, as a result, $\mathcal{G}_l = \widehat{\mathcal{S}}_2$.

Lemma 13 (Generalization of [51, Lemma 2]): For $i = 1, 2, \ldots, l + 1$, it holds

$$f(\mathcal{G}_{i-1}) - f(\mathcal{G}_i) \geq \frac{\gamma_f c(s_i)}{b^{\star}} (f(\mathcal{G}_{i-1}) - f(\mathcal{S}^{\star})).$$

Proof of Lemma 13: Due to the monotonicity of the cost function f in (10) (see Lemma 10),

$$\begin{aligned} f(\mathcal{G}_{i-1}) - f(\mathcal{S}^{\star}) &\leq f(\mathcal{G}_{i-1}) - f(\mathcal{S}^{\star} \cup \mathcal{G}_{i-1}) \\ &= f(\mathcal{G}_{i-1}) - f[(\mathcal{S}^{\star} \setminus \mathcal{G}_{i-1}) \cup \mathcal{G}_{i-1}]. \end{aligned}$$

Let $\{z_1, z_2, \ldots, z_m\} \triangleq \mathcal{S}^{\star} \setminus \mathcal{G}_{i-1}$, and also let

$$d_j \triangleq f(\mathcal{G}_{i-1} \cup \{z_1, z_2, \ldots, z_{j-1}\}) - f(\mathcal{G}_{i-1} \cup \{z_1, z_2, \ldots, z_j\})$$

for $j = 1, 2, \ldots, m$. Then, $f(\mathcal{G}_{i-1}) - f(\mathcal{S}^{\star}) \leq \sum_{j=1}^{m} d_j$. Now,

$$\frac{d_j}{c(z_j)} \leq \frac{f(\mathcal{G}_{i-1}) - f(\mathcal{G}_{i-1} \cup \{z_j\})}{\gamma_f c(z_j)} \leq \frac{f(\mathcal{G}_{i-1}) - f(\mathcal{G}_i)}{\gamma_f c(s_i)}$$

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where the first inequality holds due to the Definition 3 of γ_f, and the second due to the greedy rule (see Algorithm 2 line 13) and the definitions of $\mathcal{G}_i$, and s_i. Since $\sum_{j=1}^{m} c(z_j) \leq b^\star$,

$$f(\mathcal{G}_{i-1}) - f(\mathcal{S}^\star) \leq \sum_{j=1}^{m} d_j \leq b^\star \frac{f(\mathcal{G}_{i-1}) - f(\mathcal{G}_i)}{\gamma_f c(s_i)}.$$

■

Lemma 14 (Adaption of [51, Lemma 3]): For $i = 1, 2, \ldots, l+1$,

$$f(\emptyset) - f(G_i) \geq \left[1 - \prod_{j=1}^{i} \left(1 - \frac{\gamma_f c(s_j)}{b^\star}\right)\right] [f(\emptyset) - f(\mathcal{S}^\star)].$$

Proof of Lemma 14: We complete the proof inductively. For $i = 1$, we need to prove $f(\emptyset) - f(G_1) \geq \gamma_f c(s_1)/b^\star [f(\emptyset) - f(\mathcal{S}^\star)]$, which follows from Lemma 13 for $i = 1$. Then, for $i > 1$,

$$\begin{aligned}
f(\emptyset) - f(\mathcal{G}_i) &= f(\emptyset) - f(\mathcal{G}_{i-1}) + [f(\mathcal{G}_{i-1}) - f(\mathcal{G}_i)] \\
&\geq f(\emptyset) - f(\mathcal{G}_{i-1}) + \\
&\quad \frac{\gamma_f c(s_i)}{b^\star} (f(G_{i-1}) - f(\mathcal{S}^\star)) \\
&= \left(1 - \frac{\gamma_f c(s_i)}{b^\star}\right) [f(\emptyset) - f(\mathcal{G}_{i-1}]) + \\
&\quad \frac{\gamma_f c(s_i)}{b^\star} [f(\emptyset) - f(\mathcal{S}^\star)] \\
&\geq \left(1 - \frac{\gamma_f c(s_i)}{b^\star}\right) \left[1 - \prod_{j=1}^{i-1} \left(1 - \frac{\gamma_f c(s_j)}{b^\star}\right)\right] \\
&\quad [f(\emptyset) - f(\mathcal{S}^\star)] + \frac{\gamma_f c(s_i)}{b^\star} [f(\emptyset) - f(\mathcal{S}^\star)] \\
&= \left[1 - \prod_{j=1}^{i} \left(1 - \frac{\gamma_f c(s_j)}{b^\star}\right)\right] [f(\emptyset) - f(\mathcal{S}^\star)]
\end{aligned}$$

where we used Lemma 13 for the first inequality, and the induction hypothesis for the second. ■

Proof of Part (1) of Theorem 3: To prove Algorithm 1's approximation bound $\gamma_g/2(1 - e^{-\gamma_g})$, we let $b' \triangleq \sum_{j=1}^{l+1} c(s_j)$. Then,

$$\begin{aligned}
f(\emptyset) - f(\mathcal{G}_{l+1}) &\geq \left[1 - \prod_{j=1}^{l+1} \left(1 - \frac{\gamma_f c(s_j)}{b^\star}\right)\right] [f(\emptyset) - f(\mathcal{S}^\star)] \\
&\geq \left(1 - e^{-\gamma_f b'/b^\star}\right) [f(\emptyset) - f(\mathcal{S}^\star)] \\
&\geq \left(1 - e^{-\gamma_f}\right) [f(\emptyset) - f(\mathcal{S}^\star)] \qquad (26)
\end{aligned}$$

where the first inequality follows from Lemma 14, the second from Lemma 9, and in (26) from that $b'/b^\star \geq 1$ and, as a result, $e^{-\gamma_f b'/b^\star} \leq e^{-\gamma_f}$, that is, $1 - e^{-\gamma_f b'/b^\star} \geq 1 - e^{-\gamma_f}$.

Also, $f(\emptyset) - f(\widehat{\mathcal{S}}_1) \geq \gamma_f [f(\mathcal{G}_l) - f(\mathcal{G}_{l+1})]$ due to the Definition 3 of γ_g and, as a result,

$$\begin{aligned}
&\gamma_f [f(\emptyset) - f(\mathcal{G}_{l+1})] \\
&\leq f(\emptyset) - f(\widehat{\mathcal{S}}_1) + \gamma_f [f(\emptyset) - f(\mathcal{G}_l)] \\
&\leq 2 \max \left\{ f(\emptyset) - f(\widehat{\mathcal{S}}_1), \gamma_f [f(\emptyset) - f(\mathcal{G}_l)] \right\}. \qquad (27)
\end{aligned}$$

Substituting (26) in (27), and rearranging gives

$$\begin{aligned}
&\max \left\{ f(\emptyset) - f(\widehat{\mathcal{S}}_1), \gamma_f [f(\emptyset) - f(\mathcal{G}_l)] \right\} \\
&\geq \frac{\gamma_f}{2} \left(1 - e^{-\gamma_f}\right) [f(\emptyset) - f(\mathcal{S}^\star)]
\end{aligned}$$

which implies (since γ_f takes values in [0,1])

$$\begin{aligned}
&\max \left[f(\emptyset) - f(\widehat{\mathcal{S}}_1), f(\emptyset) - f(\mathcal{G}_l) \right] \\
&\geq \frac{\gamma_f}{2} \left(1 - e^{-\gamma_f}\right) [f(\emptyset) - f(\mathcal{S}^\star)]. \qquad (28)
\end{aligned}$$

Finally, the bound $\gamma_g/2(1 - e^{-\gamma_g})$ follows from (28) as the combination of the following three observations: i) $\mathcal{G}_l = \widehat{\mathcal{S}}_2$, and, as a result, $f(\emptyset) - f(\mathcal{G}_l) = f(\emptyset) - f(\widehat{\mathcal{S}}_2)$. ii) Algorithm 2 returns $\widehat{\mathcal{S}}$ such at $\widehat{\mathcal{S}} \in \arg\max_{\mathcal{S} \in \{\widehat{\mathcal{S}}_1, \widehat{\mathcal{S}}_2\}} [f(\emptyset) - f(\mathcal{S})]$ and, as a result, the previous observation, along with (28) gives

$$f(\emptyset) - f(\widehat{\mathcal{S}}) \geq \frac{\gamma_f}{2} \left(1 - e^{-\gamma_f}\right) [f(\emptyset) - f(\mathcal{S}^\star)]. \qquad (29)$$

iii) Finally, Lemma 11 implies that for any $\mathcal{S}, \mathcal{S}'$, $g(\mathcal{S}) = f(\mathcal{S}) + \mathbb{E}(\|x_1\|_{N_1}) + \sum_{t=1}^{T} \mathrm{tr}(W_t S_t)$, where $\mathbb{E}(\|x_1\|_{N_1}) + \sum_{t=1}^{T} \mathrm{tr}(W_t S_t)$ is independent of $\mathcal{S}$. As a result, for any $\mathcal{S}, \mathcal{S}' \subseteq \mathcal{V}$, then $f(\mathcal{S}) - f(\mathcal{S}') = g(\mathcal{S}) - g(\mathcal{S}')$, which implies $\gamma_f = \gamma_g$ due to Definition 3. In addition, Lemma 11 implies for any $\mathcal{S} \subseteq \mathcal{V}$ that $g(\mathcal{S}) = h[\mathcal{S}, u_{1:T}(\mathcal{S})]$ and $g^\star = g(\mathcal{S}^\star)$. Thereby, for any $\mathcal{S}$ that $f(\emptyset) - f(\mathcal{S}) = g(\emptyset) - g(\mathcal{S}) = h[\emptyset, u_{1:T}(\emptyset)] - h[\mathcal{S}, u_{1:T}(\mathcal{S})]$ and $f(\emptyset) - f(\mathcal{S}^\star) = g(\emptyset) - g(\mathcal{S}^\star) = h[\emptyset, u_{1:T}(\emptyset)] - g^\star$. Overall, (29) is written as

$$\begin{aligned}
&h[\emptyset, u_{1:T}(\emptyset)] - h[\widehat{\mathcal{S}}, u_{1:T}(\widehat{\mathcal{S}})] \geq \\
&\qquad \frac{\gamma_f}{2} \left(1 - e^{-\gamma_f}\right) \{h[\emptyset, u_{1:T}(\emptyset)] - g^\star\}
\end{aligned}$$

which implies the bound $\gamma_g/2(1 - e^{-\gamma_g})$.

It remains to prove $1 - e^{-\gamma_g c(\widehat{\mathcal{S}})/b}$:

$$\begin{aligned}
f(\emptyset) - f(\mathcal{G}_l) &\geq \left[1 - \prod_{j=1}^{l} \left(1 - \frac{\gamma_f c(s_j)}{b^\star}\right)\right] [f(\emptyset) - f(\mathcal{G}_l)] \\
&\geq \left(1 - e^{-\gamma_f c(\mathcal{G}_l)/b^\star}\right) [f(\emptyset) - f(\mathcal{S}^\star)] \\
&\geq \left(1 - e^{-\gamma_f c(\mathcal{G}_l)/b}\right) [f(\emptyset) - f(\mathcal{S}^\star)] \qquad (30)
\end{aligned}$$

where the first inequality follows from Lemma 14, the second from Lemma 9, and (30) from that $c(\mathcal{G}_l)/b^\star \geq c(\mathcal{G}_l)/b$, since $b^\star \leq b$, which implies $e^{-\gamma_f c(\mathcal{G}_l)/b^\star} \leq e^{-\gamma_f c(\mathcal{G}_l)/b}$, i.e., $1 - e^{-\gamma_f b'/b^\star} \geq 1 - e^{-\gamma_f c(\mathcal{G}_l)/b}$. The proof is completed using the observations (i)–(iii) mentioned above for $\gamma_g/2(1 - e^{-\gamma_g})$. ■

Proof of Part (2) of Theorem 3: The proof is parallel to that of [70, Th. 2]. ■

APPENDIX E
PROOF OF THEOREM 4

We consider the notation in Appendix D. Also, let $\mathcal{S}^\star$ be a solution to Problem 2, and $b^\star = c(\mathcal{S}^\star)$. Consider the computation

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of the set $\widehat{\mathcal{S}}$ in Algorithm 4, and let $\mathcal{G} \triangleq \widehat{\mathcal{S}}$ be the returned one. Let s_i be the ith element added in $\mathcal{G}$ during the ith iteration of Algorithm 4 "while loop." Finally, let $\mathcal{G}_i \triangleq \{s_1, s_2, \ldots, s_i\}$.

Lemma 15 (Adaptation of Lemma 13): For $i=1,2,\ldots,|\mathcal{G}|$,

$$f(\mathcal{G}_{i-1}) - f(G_i) \geq \frac{\gamma_f c(s_i)}{b^\star}(f(G_{i-1}) - f(\mathcal{S}^\star)).$$

Proof: The proof is parallel to Lemma 13's proof. ■

Lemma 16 (Adaptation of Lemma 14): For $i = 1,2,\ldots,|\mathcal{G}|$,

$$f(\emptyset) - f(G_i) \geq \left[1 - \prod_{j=1}^{i}\left(1 - \frac{\gamma_f c(s_j)}{b^\star}\right)\right][f(\emptyset) - f(\mathcal{S}^\star)].$$

Proof: The proof is parallel to Lemma 14's proof. ■

Proof of Part (1) of Theorem 4: We first observe (18) holds since Algorithm 3 returns $\widehat{\mathcal{S}}$ once $h[\widehat{\mathcal{S}}, u_{1:T}(\widehat{\mathcal{S}})] \leq \kappa$ is satisfied.

It remains to prove (19). Let $l \triangleq |\mathcal{G}|$; then, $\mathcal{G}_l = \mathcal{G}$, by the definition of $\mathcal{G}_i$, and from Lemma 14 for $i = l-1$,

$$\begin{aligned} f(\emptyset) - f(\mathcal{G}_{l-1}) &\geq \left[1 - \prod_{j=1}^{l-1}\left(1 - \frac{\gamma_f c(s_j)}{b^\star}\right)\right][f(\emptyset) - f(\mathcal{S}^\star)] \\ &\geq \left(1 - e^{-\gamma_f c(\mathcal{G}_{l-1})/b^\star}\right)[f(\emptyset) - f(\mathcal{S}^\star)] \quad (31) \end{aligned}$$

where (31) follows from Lemma 9. Moreover, Lemma 11 implies that for any $\mathcal{S}, \mathcal{S}'$, it is $g(\mathcal{S}) = f(\mathcal{S}) + \mathbb{E}(\|x_1\|_{N_1}) + \sum_{t=1}^{T}\text{tr}(W_t S_t)$, where $\mathbb{E}(\|x_1\|_{N_1}) + \sum_{t=1}^{T}\text{tr}(W_t S_t)$ is independent of $\mathcal{S}$, and, as a result, $f(\mathcal{S}) - f(\mathcal{S}') = g(\mathcal{S}) - g(\mathcal{S}')$, which implies $\gamma_f = \gamma_g$. Moreover, Lemma 11 implies for any $\mathcal{S} \subseteq$ that $g(\mathcal{S}) = h[\mathcal{S}, u_{1:T}(\mathcal{S})]$, and, as a result, $f(\emptyset) - f(\mathcal{G}_{l-1}) = h[\emptyset, u_{1:T}(\emptyset)] - h[\mathcal{G}_{l-1}, u_{1:T}(\mathcal{G}_{l-1})]$ and $f(\emptyset) - f(\mathcal{S}^\star) = h[\emptyset, u_{1:T}(\emptyset)] - h[\mathcal{S}^\star, u_{1:T}(\mathcal{S}^\star)]$. In sum, (31) is the same as the inequality

$$\begin{aligned} &h[\emptyset, u_{1:T}(\emptyset)] - h[\mathcal{G}_{l-1}, u_{1:T}(\mathcal{G}_{l-1})] \geq \\ &\left(1 - e^{-\gamma_g c(\mathcal{G}_{l-1})/b^\star}\right)\{h[\emptyset, u_{1:T}(\emptyset)] - h[\mathcal{S}^\star, u_{1:T}(\mathcal{S}^\star)]\} \end{aligned}$$

which, by letting $\beta \triangleq 1 - e^{-\gamma_g c(\mathcal{G}_{l-1})/b^\star}$ and rearranging, gives

$$\begin{aligned} h[\mathcal{G}_{l-1}, u_{1:T}(\mathcal{G}_{l-1})] &\leq (1-\beta)h[\emptyset, u_{1:T}(\emptyset)] + \beta h[\mathcal{S}^\star, u_{1:T}(\mathcal{S}^\star)] \\ &\leq (1-\beta)h[\emptyset, u_{1:T}(\emptyset)] + \beta\kappa, \quad (32) \end{aligned}$$

where the second inequality holds because $\mathcal{S}^\star$ is a solution to Problem 2 and, as result, $h[\mathcal{S}^\star, u_{1:T}(\mathcal{S}^\star)] \leq \kappa$. Now, we recall Algorithm 4 returns $\mathcal{G} = \mathcal{G}_l$ when for $i = l$ it is the first time $h[\mathcal{G}_i, u_{1:T}(\mathcal{G}_i)] \leq \kappa$. Therefore, $h[\mathcal{G}_{l-1}, u_{1:T}(\mathcal{G}_{l-1})] > \kappa$ and, as a result, there exists $\epsilon > 0$ such that $h[\mathcal{G}_{l-1}, u_{1:T}(\mathcal{G}_{l-1})] = (1+\epsilon)\kappa$, and (32) gives

$$\begin{aligned} &(1+\epsilon)\kappa \leq (1-\beta)h[\emptyset, u_{1:T}(\emptyset)] + \beta\kappa \Rightarrow \\ &\epsilon\kappa \leq (1-\beta)h[\emptyset, u_{1:T}(\emptyset)] - (1-\beta)\kappa \Rightarrow \\ &\epsilon\kappa \leq (1-\beta)\{h[\emptyset, u_{1:T}(\emptyset)] - \kappa\} \Rightarrow \\ &\epsilon\kappa \leq e^{-\gamma_g c(\mathcal{G}_{l-1})/b^\star}\{h[\emptyset, u_{1:T}(\emptyset)] - \kappa\} \Rightarrow \\ &\log\left(\frac{\epsilon\kappa}{h[\emptyset, u_{1:T}(\emptyset)] - \kappa}\right) \leq -\gamma_g c(\mathcal{G}_{l-1})/b^\star \Rightarrow \\ &c(\mathcal{G}_{l-1}) \leq \frac{1}{\gamma_g}\log\left(\frac{h[\emptyset, u_{1:T}(\emptyset)] - \kappa}{\epsilon\kappa}\right) b^\star \Rightarrow \\ &c(\mathcal{G}) \leq c(s_l) + \frac{1}{\gamma_g}\log\left(\frac{h[\emptyset, u_{1:T}(\emptyset)] - \kappa}{\epsilon\kappa}\right) b^\star \end{aligned}$$

where the latter holds since $\mathcal{G} = \mathcal{G}_{l-1} \cup \{s_l\}$, due to the definitions of $\mathcal{G}$, $\mathcal{G}_{l-1}$, and s_l, and since $c(\mathcal{G}) = c(\mathcal{G}_{l-1}) + c(s_l)$. Finally, since the definition of ϵ implies $\epsilon\kappa = h[\mathcal{G}_{l-1}, u_{1:T}(\mathcal{G}_{l-1})] - \kappa$, and the definition of $\mathcal{G}$ is $\mathcal{G} = \widehat{\mathcal{S}}$, the proof of (18) is complete. ■

Proof of Part (2) of Theorem 4: The proof is similar to the proof of part (2) of Theorem 3. ■

Appendix F
Proof of Theorem 5

We complete the proof by first deriving a lower bound for the numerator of γ_g, and then, by deriving an upper bound for the denominator γ_g. We use the following notation: $c \triangleq \mathbb{E}(x_1^\mathsf{T} N_1 x_1) + \sum_{t=1}^{T}\text{tr}(W_t S_t)$, and for any $\mathcal{S}$, and time $t = 1,2,\ldots,T$, $f_t(\mathcal{S}) \triangleq \text{tr}(\Theta_t \Sigma_{t|t}(\mathcal{S}))$. Then, $g(\mathcal{S}) = c + \sum_{t=1}^{T} f_t(\mathcal{S})$, due to (24) in Lemma 11.

a) Lower Bound for the Numerator of γ_g: The numerator of γ_g has the form $\sum_{t=1}^{T}[f_t(\mathcal{S}) - f_t(\mathcal{S}\cup\{v\})]$, for some $\mathcal{S}$, and $v \in \mathcal{V}$. We now lower bound each $f_t(\mathcal{S}) - f_t(\mathcal{S}\cup\{v\})$: from (22) in Lemma 5, observe

$$\Sigma_{t|t}(\mathcal{S}\cup\{v\}) = [\Sigma_{t|t-1}^{-1}(\mathcal{S}\cup\{v\}) + \textstyle\sum_{i\in\mathcal{S}\cup\{v\}} \bar{C}_{i,t}^\mathsf{T}\bar{C}_{i,t}]^{-1}.$$

Define $\Omega_t = \Sigma_{t|t-1}^{-1}(\mathcal{S}) + \sum_{i\in\mathcal{S}}^{T}\bar{C}_{i,t}^\mathsf{T}\bar{C}_{i,t}$ and $\bar{\Omega}_t = \Sigma_{t|t-1}^{-1}(\mathcal{S}\cup\{v\}) + \sum_{i\in\mathcal{S}}^{T}\bar{C}_{i,t}^\mathsf{T}\bar{C}_{i,t}$; using Lemma 3,

$$\begin{aligned} f_t(\mathcal{S}\cup\{v\}) &= \text{tr}\left(\Theta_t\bar{\Omega}_t^{-1}\right) \\ &\quad -\text{tr}\left(\Theta_t\bar{\Omega}_t^{-1}\bar{C}_{v,t}^\mathsf{T}(I + \bar{C}_{v,t}\bar{\Omega}_t^{-1}\bar{C}_{v,t}^\mathsf{T})^{-1}\bar{C}_{v,t}\bar{\Omega}_t^{-1}\right). \end{aligned}$$

Therefore, for any time $t \in \{1,2\ldots,T\}$,

$$\begin{aligned} &f_t(\mathcal{S}) - f_t(\mathcal{S}\cup\{v\}) \\ &= \text{tr}\left(\Theta_t\Omega_t^{-1}\right) - \text{tr}\left(\Theta_t\bar{\Omega}_t^{-1}\right) \\ &+ \text{tr}\left(\Theta_t\bar{\Omega}_t^{-1}\bar{C}_{v,t}^\mathsf{T}(I + \bar{C}_{v,t}\bar{\Omega}_t^{-1}\bar{C}_{v,t}^\mathsf{T})^{-1}\bar{C}_{v,t}\bar{\Omega}_t^{-1}\right) \\ &\geq \text{tr}\left(\Theta_t\bar{\Omega}_t^{-1}\bar{C}_{v,t}^\mathsf{T}(I + \bar{C}_{v,t}\bar{\Omega}_t^{-1}\bar{C}_{v,t}^\mathsf{T})^{-1}\bar{C}_{v,t}\bar{\Omega}_t^{-1}\right) \quad (33) \end{aligned}$$

where (33) holds because $\text{tr}(\Theta_t\Omega_t^{-1}) \geq \text{tr}(\Theta_t\bar{\Omega}_t^{-1})$. In particular, $\text{tr}(\Theta_t\Omega_t^{-1}) \geq \text{tr}(\Theta_t\bar{\Omega}_t^{-1})$ is implied as follows. Lemma 6 implies $\Sigma_{1|1}(\mathcal{S}) \succeq \Sigma_{1|1}(\mathcal{S}\cup\{v\})$. Then, Corollary 8 implies $\Sigma_{t|t-1}(\mathcal{S}) \succeq \Sigma_{t|t-1}(\mathcal{S}\cup\{v\})$, and as a result, Lemma 1 implies $\Sigma_{t|t-1}(\mathcal{S})^{-1} \preceq \Sigma_{t|t-1}(\mathcal{S}\cup\{v\})^{-1}$. Now, $\Sigma_{t|t-1}(\mathcal{S})^{-1} \preceq \Sigma_{t|t-1}(\mathcal{S}\cup\{v\})^{-1}$ and the definition of Ω_t and of $\bar{\Omega}_t$ imply $\Omega_t \preceq \bar{\Omega}_t$. Next, Lemma 1 implies $\Omega_t^{-1} \succeq \bar{\Omega}_t^{-1}$. As a result, since also Θ_t is a symmetric matrix, Lemma 4 gives the desired inequality $\text{tr}(\Theta_t\Omega_t^{-1}) \geq \text{tr}(\Theta_t\bar{\Omega}_t^{-1})$.

Continuing from (33), we have

$$\begin{aligned} &f_t(\mathcal{S}) - f_t(\mathcal{S}\cup\{v\}) \\ &\geq \text{tr}\left(\bar{C}_{v,t}\bar{\Omega}_t^{-1}\Theta_t\bar{\Omega}_t^{-1}\bar{C}_{v,t}^\mathsf{T}(I + \bar{C}_{v,t}\bar{\Omega}_t^{-1}\bar{C}_{v,t}^\mathsf{T})^{-1}\right) \\ &\geq \lambda_{\min}((I + \bar{C}_{v,t}\bar{\Omega}_t^{-1}\bar{C}_{v,t}^\mathsf{T})^{-1})\text{tr}\left(\bar{C}_{v,t}\bar{\Omega}_t^{-1}\Theta_t\bar{\Omega}_t^{-1}\bar{C}_{v,t}^\mathsf{T}\right) \quad (34) \end{aligned}$$

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where (34) holds due to Lemma 2. From (34), we have

$$\begin{aligned} &f_t(\mathcal{S}) - f_t(\mathcal{S}\cup\{v\}) \geq \\ &= \lambda_{\max}^{-1}(I + \bar{C}_{v,t}\bar{\Omega}_t^{-1}\bar{C}_{v,t}^{\mathsf{T}})\mathrm{tr}\left(\bar{C}_{v,t}\bar{\Omega}_t^{-1}\Theta_t\bar{\Omega}_t^{-1}\bar{C}_{v,t}^{\mathsf{T}}\right) \\ &\geq \lambda_{\max}^{-1}(I + \bar{C}_{v,t}\Sigma_{t|t}(\emptyset)\bar{C}_{v,t}^{\mathsf{T}})\mathrm{tr}\left(\bar{C}_{v,t}\bar{\Omega}_t^{-1}\Theta_t\bar{\Omega}_t^{-1}\bar{C}_{v,t}^{\mathsf{T}}\right) \\ &= \lambda_{\max}^{-1}(I + \bar{C}_{v,t}\Sigma_{t|t}(\emptyset)\bar{C}_{v,t}^{\mathsf{T}})\mathrm{tr}\left(\Theta_t\bar{\Omega}_t^{-1}\bar{C}_{v,t}^{\mathsf{T}}\bar{C}_{v,t}\bar{\Omega}_t^{-1}\right) \quad (35) \end{aligned}$$

where we used $\bar{\Omega}_t^{-1} \preceq \Sigma_{t|t}(\emptyset)$, which holds since: $\bar{\Omega}_t$ implies $\bar{\Omega}_t \succeq \Sigma_{t|t-1}^{-1}(\mathcal{S}\cup\{v\})$, and as a result, from Lemma 1 $\bar{\Omega}_t^{-1} \preceq \Sigma_{t|t-1}(\mathcal{S}\cup\{v\})$. In addition, Corollary 8 and $\Sigma_{1|1}(\mathcal{S}\cup\{v\}) \preceq \Sigma_{1|1}(\emptyset)$, which holds due to Lemma 6, imply $\Sigma_{t|t-1}(\mathcal{S}\cup\{v\}) \preceq \Sigma_{t|t-1}(\emptyset)$. Finally, from (22) in Lemma 5, $\Sigma_{t|t-1}(\emptyset) = \Sigma_{t|t}(\emptyset)$. Overall, $\bar{\Omega}_t^{-1} \preceq \Sigma_{t|t}(\emptyset)$.

Consider $t' \in \{1, 2\ldots, T\}$ such that $\bar{\Omega}_{t'}^{-1}\bar{C}_{v,t'}^{\mathsf{T}}\bar{C}_{v,t'}\bar{\Omega}_{t'}^{-1} \preceq \bar{\Omega}_t^{-1}\bar{C}_{v,t}^{\mathsf{T}}\bar{C}_{v,t}\bar{\Omega}_t^{-1}$, for any $t = 1, \ldots, T$. Also, let $\Phi \triangleq \bar{\Omega}_{t'}^{-1}\bar{C}_{v,t'}^{\mathsf{T}}\bar{C}_{v,t'}\bar{\Omega}_{t'}^{-1}$, and $l \triangleq \min_{t\in\{1,2\ldots,T\},v\in\mathcal{V}} \lambda_{\max}^{-1}(I + \bar{C}_{v,t}\Sigma_{t|t}(\emptyset)\bar{C}_{v,t}^{\mathsf{T}})$. Summing (35) across all $t \in \{1, 2\ldots, T\}$, and using Lemmata 2 and 4, we have

$$\begin{aligned} g(\mathcal{S}) - g(\mathcal{S}\cup\{v\}) &\geq l\sum_{t=1}^{T}\mathrm{tr}\left(\Theta_t\bar{\Omega}_t^{-1}\bar{C}_{v,t}^{\mathsf{T}}\bar{C}_{v,t}\bar{\Omega}_t^{-1}\right) \\ &\geq l\lambda_{\min}\left(\sum_{t=1}^{T}\Theta_t\right)\mathrm{tr}\left(\Phi\right) > 0 \end{aligned}$$

which is nonzero because $\sum_{t=1}^{T}\Theta_t \succ 0$ and Φ is a nonzero positive semidefinite matrix.

Finally, we lower bound $\mathrm{tr}(\Phi)$, using Lemma 2:

$$\begin{aligned} \mathrm{tr}\left(\Phi\right) &= \mathrm{tr}\left(\bar{\Omega}_{t'}^{-1}\bar{C}_{v,t'}^{\mathsf{T}}\bar{C}_{v,t'}\bar{\Omega}_{t'}^{-1}\right) \\ &\geq \lambda_{\min}(\bar{\Omega}_{t'}^{-2})\mathrm{tr}\left(\bar{C}_{v,t'}^{\mathsf{T}}\bar{C}_{v,t'}\right) \\ &\geq \lambda_{\min}^{2}(\Sigma_{t'|t'}(\mathcal{V}))\mathrm{tr}\left(\bar{C}_{v,t'}^{\mathsf{T}}\bar{C}_{v,t'}\right) \quad (36) \end{aligned}$$

where (36) holds because $\bar{\Omega}_{t'}^{-1} \succeq \Sigma_{t'|t'}(\mathcal{V})$. Particularly, $\bar{\Omega}_{t'}^{-1} \succeq \Sigma_{t'|t'}(\mathcal{S}\cup\{v\})$ is derived by applying Lemma 1 to $\bar{\Omega}_{t'} \preceq \bar{\Omega}_{t'} + \bar{C}_{v,t}^{\mathsf{T}}\bar{C}_{v,t}^{\mathsf{T}} = \Sigma_{t'|t'}^{-1}(\mathcal{S}\cup\{v\})$, where the equality holds by the definition of $\bar{\Omega}_{t'}$. In addition, due to Lemma 6, $\Sigma_{1|1}(\mathcal{S}\cup\{v\}) \succeq \Sigma_{1|1}(\mathcal{V})$, and as a result, from Corollary 7, $\Sigma_{t'|t'}(\mathcal{S}\cup\{v\}) \succeq \Sigma_{t'|t'}(\mathcal{V})$. Overall, $\bar{\Omega}_{t'}^{-1} \succeq \Sigma_{t'|t'}(\mathcal{V})$ holds.

b) Upper Bound for the Denominator of γ_g: The proof follows similar ideas as mentioned above, and is omitted (for a complete proof, see [64, Proof of Th. 5]).

Appendix G
Proof of Theorem 6

Lemma 17 (System-level condition for near-optimal co-design): Let N_1 be defined as in (9). The control policy $u_{1:T}^{\circ} \triangleq (0, 0, \ldots, 0)$ is suboptimal for the LQG problem in (21) for all nonzero initial conditions x_1 if and only if

$$\sum_{t=1}^{T} A_1^{\mathsf{T}}\cdots A_t^{\mathsf{T}}Q_tA_t\cdots A_1 \succ N_1. \quad (37)$$

Proof of Lemma 17: For any x_1, (24) in Lemma 11 implies for (21) as

$$\min_{u_{1:T}} \sum_{t=1}^{T}[\|x_{t+1}\|_{Q_t}^2 + \|u_t(x_t)\|_{R_t}^2]\big|_{\Sigma_{t|t}=W_t=0} = x_1^{\mathsf{T}}N_1x_1 \quad (38)$$

since $\mathbb{E}(\|x_1\|_{N_1}^2) = x_1^{\mathsf{T}}N_1x_1$, because x_1 is known ($\Sigma_{1|1} = 0$), and $\Sigma_{t|t}$ and W_t are zero. In addition, for $u_{1:T} = (0, 0, \ldots, 0)$, the objective function in (21) is

$$\begin{aligned} &\textstyle\sum_{t=1}^{T}[\|x_{t+1}\|_{Q_t}^2 + \|u_t(x_t)\|_{R_t}^2]\big|_{\Sigma_{t|t}=W_t=0} \\ &= \textstyle\sum_{t=1}^{T} x_{t+1}^{\mathsf{T}}Q_tx_{t+1} \\ &= x_1^{\mathsf{T}}\textstyle\sum_{t=1}^{T} A_1^{\mathsf{T}}A_2^{\mathsf{T}}\cdots A_t^{\mathsf{T}}Q_tA_tA_{t-1}\cdots A_1x_1 \end{aligned} \quad (39)$$

since $x_{t+1} = A_tA_{t-1}\cdots A_1x_1$ when all $u_1, \ldots, u_T$ are zero.

From (38) and (39), we have that $x_1^{\mathsf{T}}N_1x_1 < x_1^{\mathsf{T}}\sum_{t=1}^{T} A_1^{\mathsf{T}}A_2^{\mathsf{T}}\cdots A_t^{\mathsf{T}}Q_tA_tA_{t-1}\cdots A_1x_1$ holds for any nonzero x_1 if and only if $N_1 \prec \sum_{t=1}^{T} A_1^{\mathsf{T}}\cdots A_t^{\mathsf{T}}Q_tA_tA_{t-1}\cdots A_1$. ■

Lemma 18: $\Theta_t = A_t^{\mathsf{T}}S_tA_t + Q_{t-1} - S_{t-1}$, for $t = 1, \ldots, T$.

Proof of Lemma 18: Using the Woobury identity in Lemma 3, and the notation in (9), $N_t = A_t^{\mathsf{T}}(S_t^{-1} + B_tR_t^{-1}B_t^{\mathsf{T}})^{-1}A_t = A_t^{\mathsf{T}}S_tA_t - \Theta_t$. The latter, gives $\Theta_t = A_t^{\mathsf{T}}S_tA_t - N_t$. In addition, from (9), $-N_t = Q_{t-1} - S_{t-1}$, since $S_t = Q_t + N_{t+1}$. ■

Lemma 19: $\sum_{t=1}^{T} A_1^{\mathsf{T}}A_2^{\mathsf{T}}\cdots A_t^{\mathsf{T}}Q_tA_tA_{t-1}\cdots A_1 \succ N_1$ if and only if $\sum_{t=1}^{T} A_1^{\mathsf{T}}A_2^{\mathsf{T}}\cdots A_{t-1}^{\mathsf{T}}\Theta_tA_{t-1}A_{t-2}\cdots A_1 \succ 0$.

Proof: For $i = t-1, \ldots, 1$, we premultiply and postmultiply the identity in Lemma 18 with A_i^{T} and A_i, respectively:

$$\begin{aligned} &\Theta_t = A_t^{\mathsf{T}}S_tA_t + Q_{t-1} - S_{t-1} \Rightarrow \\ &A_{t-1}^{\mathsf{T}}\Theta_tA_{t-1} = A_{t-1}^{\mathsf{T}}A_t^{\mathsf{T}}S_tA_tA_{t-1} + A_{t-1}^{\mathsf{T}}Q_{t-1}A_{t-1} - \\ &\quad A_{t-1}^{\mathsf{T}}S_{t-1}A_{t-1} \Rightarrow \\ &A_{t-1}^{\mathsf{T}}\Theta_tA_{t-1} = A_{t-1}^{\mathsf{T}}A_t^{\mathsf{T}}S_tA_tA_{t-1} + A_{t-1}^{\mathsf{T}}Q_{t-1}A_{t-1} - \\ &\quad \Theta_{t-1} + Q_{t-2} - S_{t-2} \Rightarrow \\ &\Theta_{t-1} + A_{t-1}^{\mathsf{T}}\Theta_tA_{t-1} = A_{t-1}^{\mathsf{T}}A_t^{\mathsf{T}}S_tA_tA_{t-1} + \\ &\quad A_{t-1}^{\mathsf{T}}Q_{t-1}A_{t-1} + Q_{t-2} - S_{t-2} \Rightarrow \\ &\ldots \Rightarrow \\ &\quad \ldots + A_2^{\mathsf{T}}Q_2A_2 + Q_1 - S_1 \Rightarrow \\ &\Theta_1 + A_1^{\mathsf{T}}\Theta_2A_1 + \ldots + A_1^{\mathsf{T}}\cdots A_{t-1}^{\mathsf{T}}\Theta_tA_{t-1}\cdots A_1 = \\ &A_1^{\mathsf{T}}\cdots A_t^{\mathsf{T}}S_tA_t\cdots A_1 + A_1^{\mathsf{T}}\cdots A_{t-1}^{\mathsf{T}}Q_{t-1}A_{t-1}\cdots A_1 + \\ &\quad \ldots + A_1^{\mathsf{T}}Q_1A_1 - N_1 \Rightarrow \\ &\textstyle\sum_{t=1}^{T} A_1^{\mathsf{T}}\cdots A_{t-1}^{\mathsf{T}}\Theta_tA_{t-1}\cdots A_1 = \\ &\quad \textstyle\sum_{t=1}^{T} A_1^{\mathsf{T}}\cdots A_t^{\mathsf{T}}Q_tA_t\cdots A_1 - N_1. \end{aligned}$$

■

Lemma 20: Consider for any $t = 1, 2, \ldots, T$ that A_t is invertible. $\sum_{t=1}^{T} A_1^{\mathsf{T}}A_2^{\mathsf{T}}\cdots A_{t-1}^{\mathsf{T}}\Theta_tA_{t-1}A_{t-2}\cdots A_1 \succ 0$ if and only if $\sum_{t=1}^{T}\Theta_t \succ 0$.

Proof of Lemma 20: Let $U_t = A_{t-1}A_{t-2}\cdots A_1$.

We first prove that for any nonzero vector z, if it is $\sum_{t=1}^{T} A_1^{\mathsf{T}}A_2^{\mathsf{T}}\cdots A_{t-1}^{\mathsf{T}}\Theta_tA_{t-1}A_{t-2}\cdots A_1 \succ 0$, then $\sum_{t=1}^{T} z^{\mathsf{T}}\Theta_tz > 0$. Particularly, since U_t is invertible,—because for any $t \in \{1, 2, \ldots, T\}$, A_t is,—

$$\begin{aligned} \textstyle\sum_{t=1}^{T} z^{\mathsf{T}}\Theta_tz &= \textstyle\sum_{t=1}^{T} z^{\mathsf{T}}U_t^{-\top}U_t^{\mathsf{T}}\Theta_tU_tU_t^{-1}z \\ &= \textstyle\sum_{t=1}^{T}\mathrm{tr}\left(\phi_t\phi_t^{\mathsf{T}}U_t^{\mathsf{T}}\Theta_tU_t\right) \end{aligned} \quad (40)$$

where we let $\phi_t \triangleq U_t^{-1}z$. Consider a time t' such that for any time $t \in \{1, 2\ldots, T\}$, $\phi_{t'}\phi_{t'}^{\mathsf{T}} \preceq \phi_t\phi_t^{\mathsf{T}}$. From (40), using Lem-

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mata 2 and 4,

$$\sum_{t=1}^{T} z^{\mathsf{T}}\Theta_t z \geq \sum_{t=1}^{T} \mathrm{tr}\left(\phi_{t'}\phi_{t'}^{\mathsf{T}} U_t^{\mathsf{T}}\Theta_t U_t\right) = \|\phi_{t'}\|_2^2 \lambda_{\min}\left(\sum_{t=1}^{T} U_t^{\mathsf{T}}\Theta_t U_t\right) > 0.$$

We finally prove that for any nonzero vector z, if $\sum_{t=1}^{T}\Theta_t \succ 0$, then $\sum_{t=1}^{T} zA_1^{\mathsf{T}}\cdots A_{t-1}^{\mathsf{T}}\Theta_t A_{t-1}\cdots A_1 z \succ 0$:

$$\sum_{t=1}^{T} z^{\mathsf{T}} U_t^{\mathsf{T}}\Theta_t U_t z = \sum_{t=1}^{T} \mathrm{tr}\left(\xi_t^{\mathsf{T}}\Theta_t\xi_t\right) \tag{41}$$

where we let $\xi_t \triangleq U_t z$. Consider time t' such that for any time $t \in \{1,\ldots,T\}$, $\xi_{t'}\xi_{t'}^{\mathsf{T}} \preceq \xi_t\xi_t^{\mathsf{T}}$. From (40), we have

$$\sum_{t=1}^{T} \mathrm{tr}\left(\xi_t^{\mathsf{T}}\Theta_t\xi_t\right) \geq \mathrm{tr}\left(\xi_{t'}\xi_{t'}^{\mathsf{T}}\sum_{t=1}^{T}\Theta_t\right) = \|\xi_{t'}\|_2^2\lambda_{\min}\left(\sum_{t=1}^{T}\Theta_t\right) > 0.$$

■

Proof of Theorem 6: Theorem 6 follows from the sequential application of Lemmata 17, 19, and 20. ■

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