Distributed Hybrid Control for Multiple-Pursuer Multiple-Evader Games*

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Abstract. Multiple-pursuer multiple-evader games raise fundamental and novel problems in control theory and robotics. In this paper, we propose a distributed solution to this problem that simultaneously addresses the discrete assignment of pursuers to evaders as well as the continuous control strategies for capturing individual evaders. The resulting hybrid control framework guarantees the mutual exclusion property of the final assignment for all initial conditions as well as capturing all evaders after exploring at most a polynomial number of assignments, dramatically reducing the combinatorial nature of purely discrete assignment problems.

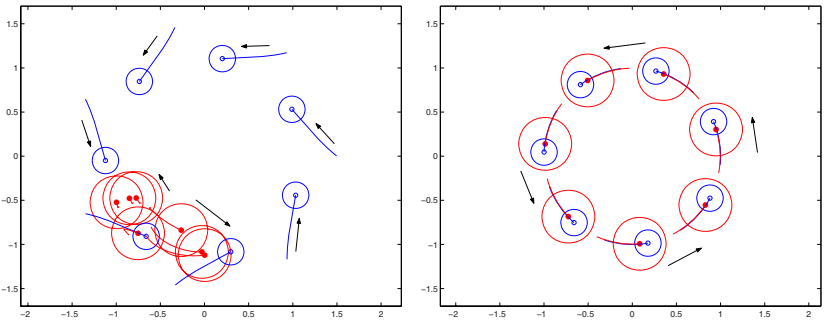
1 Background and Problem Definition

Consider n identical pursuers in a p-dimensional space $\mathbb{R}^p$ and assume kinematic models for the pursuers with, in general, unbounded velocities. Consider, further, $m \geq n$ evaders with velocities that can be time varying but are assumed to be bounded for all time. We then say that pursuer i can *capture* evader k if there exists a finite time instant $T > 0$ such that pursuer i is in a sufficiently small neighborhood of evader k, for all time $t > T$. The time instant that every pursuer has captured a distinct evader corresponds to the *termination of the game* and the notion of capturing the evaders becomes equivalent to that of assigning evaders to pursuers.

Pursuit-evasion games can be classified in continuous and purely discrete games. Continuous games explicitly model the physical motion and constraints of the players [1], [2], [3] and often assume worst case motion for the evaders [1], [2]. Studying optimality of such strategies involves numerically solving the computationally challenging Hamilton-Jacobi-Isaacs partial differential equations. On the other hand, discrete games are either played in purely discrete environments such as graphs [4], [5], or in continuous environments disregarding though any physical dynamics of the players [6], [7]. Such models may result in discrete strategies which are dynamically infeasible. Recently, however, hybrid control

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(a) Intermediate Configuration. (b) Final Configuration.

Fig. 1. Simulation for $n = 7$ pursuers and $m = 7$ evaders

has been used to bridge the gap between the discrete and continuous games and model, more realistic, pursuers with limited sensing and communication capabilities. Approaches involve visibility-based [8] and probabilistic [9], [10] games. Closely related to the topics discussed in this paper is also multiple-target tracking by sensor networks [11], [12] which, however, focuses more on the sensing and estimation problem of tracking rather than the actuation and control.

In this paper, we propose a novel distributed approach to the multiple-pursuer multiple-evader game, which is inspired by our previous work on dynamic assignment for stationary targets [13], [14]. Unlike *centralized* or *off-line* approaches [15], [16], [17], that decouple the assignment and navigation problems and focus on designing control laws for every pursuer to capture a preassigned evader, we *simultaneously* address the discrete assignment of pursuers to evaders as well as the continuous control strategies for tracking and capturing the individual evaders. The resulting hybrid controller for every pursuer consists of both local coordination protocols [14], [18] guaranteeing that distinct evaders are captured by distinct pursuers, and single-pursuer single-evader time-varying potential fields ensuring convergence of the tracking error to any neighborhood of zero. Composition of the hybrid controllers for all pursuers results in a highly efficient overall system that is illustrated in nontrivial multiple-pursuer multiple-evader games (Figure 1).

The main contribution of our approach is in the distributed coordination protocols that enable nontrivial multiple-pursuer multiple-evader behavior based on already available results for single-pursuer single-evader games. In particular, under the assumption that the pursuers have knowledge of the evaders' locations, the proposed nearest neighbor coordination scheme leads to a dynamically determined final assignment [14] which is shown to have the desired mutual exclusion property for all initial conditions. Furthermore, the overall coordination system is shown to have at most polynomial complexity, i.e., at most a polynomial number of transitions, despite the exponential growth of the number of assignments with respect to the number of pursuers.

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