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# Noncooperative Conflict Resolution [1]

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**Abstract**

Next Generation Air Traffic Management will allow for the possibility of *free flight*, in which each aircraft chooses its own optimal route, altitude and speed. In a free flight environment, the trajectories of several aircraft may be conflicting, in which case aircraft may or may not cooperate in resolving the conflict. This paper presents a method for noncooperative conflict resolution in which each aircraft develops a resolution strategy for the worst possible actions (within known bounds) of the other aircraft. The resolution strategy is based on noncooperative game theory: the solution to the game partitions the state space of each aircraft into safe and unsafe sets which represent level sets of the Hamilton-Jacobi equation, and the control strategies are abstracted into discrete protocols within each partition. Two examples of conflict resolution using speed and heading changes are worked out in detail.

## 1 Introduction

An area of great interest in the aviation community is that of *free flight* for future air traffic systems [1]. In a free flight environment, pilots would have the freedom to select their path and speed in real time, using, for example, Global Positioning Systems (GPS) as navigational tools instead of following indirect routes over the current Very High Frequency Omnidirectional Range (VOR) transmitters. Communication between aircraft would exist in the near future with the proposed Automatic Dependence Surveillance-Broadcast (ADS-B) [2], in which each aircraft broadcasts to all other aircraft in its vicinity its current state as well as intent in the form of its next two proposed way points. State information about surrounding aircraft could also be provided by on-board radar. In order to maintain the required standards of safety in free flight airspace, automatic conflict detection and resolution algorithms are vital. Sophisticated algorithms which predict and automatically resolve conflicts would be used either on the ground or on-board, either as advisories or as part of the flight management system of each aircraft. Current research endeavors along this direction include [3, 4, 5, 6, 7].

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In our work, we strive for an approach and a modeling scheme for which we can verify the safety of the conflict resolution method. We present in [8] a decentralized architecture for Air Traffic Management to use as a framework for our conflict resolution algorithms. In this framework, aircraft in en-route airspace are allowed to self-optimize in the spirit of free flight and coordinate with neighboring aircraft in order to resolve potential conflicts.

In the current paper we develop a solution for *noncooperative* conflicts, meaning that aircraft do not collaborate in resolving the conflict. We assume that each aircraft knows the current state of every other aircraft involved in a potential conflict, but that the intent of every other aircraft is unknown. This situation occurs when one of the aircraft, a general aviation aircraft for example, is not equipped with sophisticated avionics, or when there is a communication malfunction between aircraft. In this case, sensing using on-board radar is still able to provide estimates of the state variables of the surrounding aircraft, as well as the aircraft-types, which in turn would provide bounds on their aerodynamic capabilities. A natural framework for this type of problem in which many aircraft have conflicting objectives is zero-sum noncooperative dynamic game theory. The history of dynamic games is rich: the study of differential equations in game theory was first motivated by military problems in the U.S. Air Force (aircraft dogfights, target missiles) and was initially developed by Rand Corp. researcher Isaacs in the 1940's and 50's [9]. An excellent modern reference is [10], and our motivation for this work is that of [11], in which game theoretic methods were used to prove safety of a set of maneuvers in Intelligent Vehicle Highway Systems.

In this paper, we use the development of Isaacs to calculate the *safe* regions of operation, in state and control space, for each aircraft involved in the conflict. We use the saddle solution to the game to delineate the state space into safe and unsafe regions, but we require that each aircraft operate according to the saddle solution *only on the boundary of its safe region.* As such, our control scheme is called *least restrictive*, because within its safe region of operation, the aircraft may design its trajectory to optimize over other criteria, such as fuel efficiency or minimal deviation from route. The implicit switching control law derived from the saddle so-

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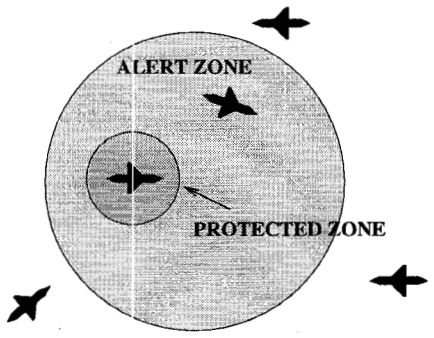



**Figure 1:** Alert zone and protected zone.

lution to the game may be modeled by finite automata with differential equations associated with each state, and the resultant hybrid system is safe by design. Our safe set computation gives the same geometric solution to that of [12], yet we work in the context of steady state solutions to the Hamilton-Jacobi equation.

The algorithms of this paper do not require any additional structure to the airspace than what currently exists. Each aircraft is surrounded by two virtual *hockey pucks*, the *protected zone* and the *alert zone* [2], shown in Figure 1. A conflict between two aircraft occurs if the protected zones of the aircraft overlap. The radius and the height of the en-route protected zone is currently 5 nautical miles and 2,000 ft respectively. The size of the alert zone depends on various factors including airspeed, altitude, accuracy of sensing equipment, traffic situation, aircraft performance and average human and system response times. The alert zone should be large enough to allow a comfortable system response but also small enough in order to avoid unnecessary conflicts. For the algorithms in this paper, we assume a two-dimensional alert zone although three dimensional extensions of the algorithms are conceptually similar. The safe operating region for each aircraft is thus the largest controlled invariant subset of the complement of the protected zones of the other aircraft involved in the conflict.

In this paper, we present our solution for two aircraft: section 2 develops the game theoretic method, and section 3 presents two examples of conflict resolution in the horizontal plane. The model used is a relative kinematic model for two aircraft, aircraft 1 and aircraft 2, which describes the motion of aircraft 2 with respect to aircraft 1:

$$\begin{aligned} \dot{x}_r &= -v_1 + v_2 \cos\phi_r + \omega_1 y_r \\ \dot{y}_r &= v_2 \sin\phi_r - \omega_1 x_r \\ \dot{\phi}_r &= \omega_2 - \omega_1 \end{aligned} \tag{1}$$

in which $(x_r, y_r, \phi_r)$ is the relative position and orientation of aircraft 2 with respect to aircraft 1, and $v_i$ and $\omega_i$ are the linear and angular velocities of each aircraft. The reader is referred to [13] for a complete derivation

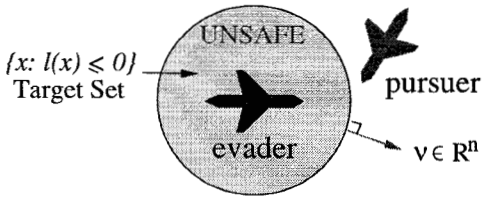



**Figure 2:** The evader and pursuer, with Target set and its outward pointing normal $\nu$

of this model, as well as for the proof of Proposition 1 in Section 3.2. Our cooperative solutions for more than two aircraft, when communication is assumed between the aircraft, are presented in [14].

## 2 Game Theoretic Approach

First, we describe our noncooperative conflict resolution design philosophy on a general relative configuration model in $\mathbb{R}^n$. Consider the system

$$\dot{x} = f(x, u, d) \qquad x(t) = x \tag{2}$$

where $x \in \mathbb{R}^n$ describes the relative configuration of one of the aircraft with respect to the other, $u \in \mathcal{U}$ is the control input of one agent, and $d \in \mathcal{D}$ is the control of the other agent. We assume that the system starts at state $x$ at initial time $t$. Both $\mathcal{U}$ and $\mathcal{D}$ are known sets, but whereas the control input $u$ may be chosen by the designer, the disturbance $d$ is unknown. The goal is to maintain safe operation of the system (2), meaning that the system trajectories do not enter a prespecified unsafe region of the state space, called the Target set and denoted $T$ with boundary $\partial T$. We assume that there exists a differentiable function $l(x)$ so that $T = \{x \in \mathbb{R}^n \mid l(x) \leq 0\}$ and $\partial T = \{x \in \mathbb{R}^n \mid l(x) = 0\}$. In this paper, $T$ represents the protected zone around the aircraft at the origin of the relative axis frame (Figure 2).

Suppose that the two aircraft are conflict-prone and that they cannot cooperate to resolve conflict. Then the safest possible strategy of each aircraft is to fly a trajectory which guarantees that the minimum allowable separation with the other aircraft is maintained, *regardless* of the actions of the other aircraft. We formulate this problem as a two-person, zero-sum dynamical game of the pursuer-evader variety. Call the aircraft at the origin of the relative frame the *evader* with control input $u$, and the other aircraft the *pursuer* with control input $d$; the goal of the evader is to drive the system outside $T$ whereas the worst possible action of the pursuer is to try to drive the system into $T$. We solve the dynamical game for system (2) over the time interval $[t, t_f]$, where $t_f$ is defined as

$$t_f = \inf\{\tau \in \mathbb{R}^+ \mid x(\tau) \in T\} \tag{3}$$

with initial state $x$ at time $t$. If $t_f = \infty$, then for all possible control actions and disturbances the trajectory never enters $T$.



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The game is a variational problem without a running cost, or Lagrangian: we are interested only in whether or not the state enters $T$. The cost $J_1(x,t,u,d)$ is therefore defined as a function (only) of the terminal state:

$$J_1(x,t,u,d) = l(x(t_f)) \tag{4}$$

Given $J_1(x,t,u,d)$, we first characterize the *unsafe* portion of $\partial T$, defined as those states $x \in \partial T$ for which there exists some disturbance $d \in \mathcal{D}$ such that for all inputs $u \in \mathcal{U}$ the vector field points into $T$; the *safe* portion of $\partial T$ consists of the states $x \in \partial T$ for which there is some input $u \in \mathcal{U}$ such that for all disturbances $d \in \mathcal{D}$, the vector field points outward from $T$. More formally, we denote the outward pointing normal to $T$ as

$$\nu = \frac{\partial l}{\partial x}(x(t_f)) \tag{5}$$

as in Figure 2 which allows us to define

$$\begin{array}{l}\text{Safe part of } \partial T \ \{x \in \partial T : \exists u \forall d \ \nu^T f(x,u,d) \geq 0\} \\ \text{Unsafe part of } \partial T \ \{x \in \partial T : \forall u \exists d \ \nu^T f(x,u,d) < 0\}\end{array} \tag{6}$$

Given the above anatomy of $\partial T$, the game is won by the pursuer if the terminal state $x(t_f)$ belongs in the unsafe portion of the boundary, and is won by the evader otherwise. It is clear that the optimal control $u^* \in \mathcal{U}$ is the one which maximizes $J_1(x,t,u,d)$, and the worst disturbance $d^* \in \mathcal{D}$ is the one which minimizes $J_1(x,t,u,d)$:

$$u^* = \arg\max_{u \in \mathcal{U}} J_1(x,t,u,d) \tag{7}$$

$$d^* = \arg\min_{d \in \mathcal{D}} J_1(x,t,u,d) \tag{8}$$

The game is said to have a saddle solution if the cost $J_1^*(x,t)$ does not depend on the order in which the maximization and minimization is performed:

$$J_1^*(x,t) = \max_{u \in \mathcal{U}} \min_{d \in \mathcal{D}} J_1(x,t,u,d) = \min_{d \in \mathcal{D}} \max_{u \in \mathcal{U}} J_1(x,t,u,d) \tag{9}$$

The concept of a saddle solution is key to our computation of the safe regions of operation of the aircraft, since a solution of (2) with $u = u^*$ and $d = d^*$ represents an optimal trajectory for *each* player under the assumption that the other player plays its optimal strategy.

Safety is maintained by operating within the *safe set* of states $V_1$, which is the largest subset of $\mathbb{R}^n \backslash T$ which can be rendered invariant using inputs $u \in \mathcal{U}$ regardless of the disturbance $d \in \mathcal{D}$. We define $V_1$ as

$$V_1 = \{x \in \mathbb{R}^n \backslash T \mid \exists u \in \mathcal{U}, J_1(x,t,u,d^*) \geq 0\} \tag{10}$$

Let $\partial V_1$ denote the boundary of $V_1$. At any instant $t$, the set $\{x \in \mathbb{R}^n \backslash T \mid J_1^*(x,t) \geq 0\}$ defines the set of safe states starting from time $t$. We would like to calculate the "steady state" safe set, or the safe set of states for all $t \in (-\infty, t_f]$. For this purpose we construct the Hamilton-Jacobi equation for this system and attempt

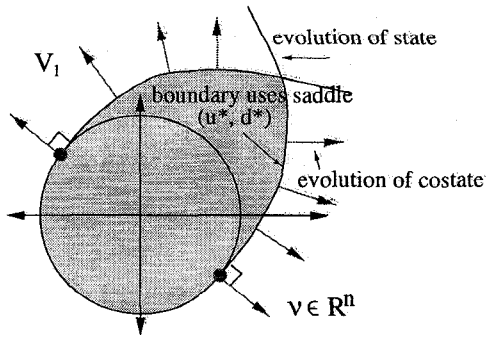



**Figure 3:** The unsafe set of states (shaded) and its complement (the safe set $V_1$)

to calculate its steady state solution. Define the Hamiltonian $H(x,p,u,d) = pf(x,u,d)$ where $p \in T^*\mathbb{R}^n$ is the costate. The optimal Hamiltonian is given by:

$$H^*(x,p) = \max_{u \in U} \min_{d \in D} H(x,p,u,d) = H(x,p,u^*,d^*) \tag{11}$$

and satisfies Hamilton's equations (provided $H^*(x,p)$ is smooth in $x$ and $p$):

$$\begin{array}{rcl}\dot{x} &=& \frac{\partial H^*}{\partial p}(x,p) \\ \dot{p} &=& -\frac{\partial H^*}{\partial x}(x,p)\end{array} \tag{12}$$

with the boundary conditions $p(t_f) = \partial l(x(t_f))/\partial x$ and $x(t_f) \in \partial T$. If $J_1^*(x,t)$ is a smooth function of $x$ and $t$, then $J_1^*(x,t)$ satisfies the Hamilton-Jacobi equation:

$$\frac{\partial J_1^*(x,t)}{\partial t} = -H^*(x, \frac{\partial J_1^*(x,t)}{\partial x}) \tag{13}$$

with boundary condition $J_1^*(x,t_f) = l(x(t_f))$. Our goal is to compute the safe set $V_1 = \{x \in \mathbb{R}^n \backslash T \mid J_1^*(x,-\infty) \geq 0\}$ where $J_1^*(x,-\infty)$ is the steady state solution of (13). However, it is difficult to guarantee that the PDE (13) has solutions for all $t \leq 0$, due to the occurrence of "shocks", ie. discontinuities in $J$ as a function of $x$. If there are no shocks in the solution of (13), we may compute $J_1^*(x,-\infty)$ by setting the left hand side of the Hamilton-Jacobi equation to zero, thus $H^*(x, \frac{\partial J_1^*(x,-\infty)}{\partial x}) = 0$ which implies that $\frac{\partial J_1^*(x,-\infty)}{\partial x}$ is normal to the vector field $f(x,u^*,d^*)$.

To compute the safe set, we can propagate the boundaries of the safe set (those points for which $l(x) = 0$ and $H^*(x,p) = 0$) backwards in time, using the Hamilton-Jacobi equation (13) to determine the safe and unsafe sets over the state space $\mathbb{R}^n$ (see Figure 3).

The set $V_1$ defines the *least restrictive control scheme* for safety. If the pursuer is inside $V_1$, any control input may be safely applied by the evader, whereas on the boundary, the only input which may be safely applied to ensure safety is $u^*$. If the pursuer is inside the unsafe set, it can eventually end up in the target set regardless of the actions of the evader. The safe set of control



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inputs associated with each state $x \in V_1$ is

$$\mathcal{U}_1(x) = \{u \in \mathcal{U} \mid J_1(x,t,u,d) \geq 0, \forall d \in \mathcal{D}\} \quad (14)$$

Since all $u \in \mathcal{U}_1$ guarantee safety from state $x$, it is advantageous to find the best control policy within this set, such as the one that minimizes deviation from the nominal trajectory which is encoded by a second cost function $J_2$, usually a quadratic function of the tracking error. To do this, we solve the optimal control problem which is *nested inside* the differential game calculation: $(\min_{u \in \mathcal{U}_1} J_2)$ subject to the differential equations which describe the aircraft motion in absolute coordinates. Additional system requirements, such as *passenger comfort*, can now be incorporated by extending the above nested chain of games and optimal control problems following the multiobjective design methodology of [15].

## 3 Constant Altitude Conflict Resolution

In this section, we apply this general framework to conflicts which occur among aircraft at the same altitude. These conflicts are resolved either by path deviations or speed variations.

### 3.1 Resolution by Angular Velocity

Let us first consider the case in which the linear velocities of both aircraft are fixed, $v_1, v_2 \in \mathbb{R}$, and the aircraft avoid conflict solely by using their angular velocities, thus $u = \omega_1$ and $d = \omega_2$, and model (1) becomes:

$$\begin{aligned} \dot{x}_r &= -v_1 + v_2 \cos\phi_r + u y_r \\ \dot{y}_r &= v_2 \sin\phi_r - u x_r \\ \dot{\phi}_r &= d - u \end{aligned} \quad (15)$$

with state variables $x_r, y_r \in \mathbb{R}$, $\phi_r \in [-\pi, \pi)$, and control and disturbance inputs $u \in \mathcal{U} = [\underline{\omega}_1, \overline{\omega}_1] \subset \mathbb{R}$, $d \in \mathcal{D} = [\underline{\omega}_2, \overline{\omega}_2] \subset \mathbb{R}$. Without loss of generality (we scale the coefficients of $u$ and $d$ if this is not met), assume that $\underline{\omega}_i = -1$ and $\overline{\omega}_i = 1$, for $i = 1, 2$.

The target set $T$ is the protected zone of the evader:

$$T = \{(x_r, y_r) \in \mathbb{R}^2, \phi_r \in [-\pi, \pi) \mid x_r^2 + y_r^2 \leq 5^2\} \quad (16)$$

which is a 5-mile-radius cylindrical block in the $(x_r, y_r, \phi_r)$ space. Thus the function $l(x)$ may be defined as $l(x) = x_r^2 + y_r^2 - 5^2$.

The optimal Hamiltonian is

$$H^*(x,p) = \max_{u \in U} \min_{d \in D} [-p_1 v_1 + p_1 v_2 \cos\phi_r + p_2 v_2 \sin\phi_r + (p_1 y_r - p_2 x_r - p_3)u + p_3 d] \quad (17)$$

Defining the *switching functions* $s_1(t)$ and $s_2(t)$, as

$$\begin{aligned} s_1(t) &= p_1(t) y_r(t) - p_2(t) x_r(t) - p_3(t) \\ s_2(t) &= p_3(t) \end{aligned} \quad (18)$$

the saddle solution $u^*, d^*$ exists when $s_1 \neq 0$ and $s_2 \neq 0$ and are calculated as

$$\begin{aligned} u^* &= sgn(s_1) \\ d^* &= -sgn(s_2) \end{aligned} \quad (19)$$

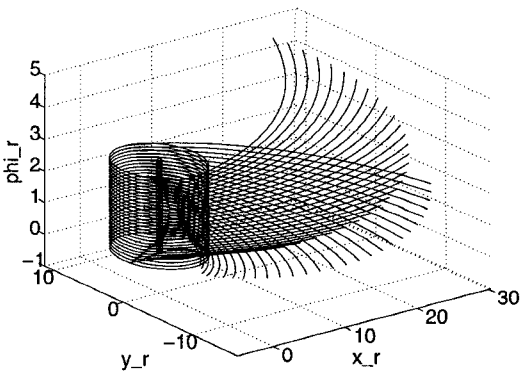



**Figure 4:** The Target set $T = \{(x_r, y_r), \phi_r \in (0, \pi) \mid x_r^2 + y_r^2 \leq 5^2\}$ (cylinder) and the boundary of the safe set $V_1$ for $t \leq t_f$ until the first switch in either $s_1(t)$ or $s_2(t)$. The unsafe set is enclosed by the boundary.

The equations for $\dot{p}$ are obtained through (12) and are

$$\begin{aligned} \dot{p}_1 &= u^* p_2 \\ \dot{p}_2 &= -u^* p_1 \\ \dot{p}_3 &= p_1 v_2 \sin\phi_r - p_2 v_2 \cos\phi_r \end{aligned} \quad (20)$$

with $p(t_f) = (x_r, y_r, 0)^T = \nu$, the outward pointing normal to $\partial T$ at any point $(x_r, y_r, \phi_r)$ on $\partial T$. The safe and unsafe portions of $\partial T$ are calculated using equations (6) with $\nu = (x_r, y_r, 0)^T$. Thus, those $(x_r, y_r, \phi_r)$ on $\partial T$ for which

$$-v_1 x_r + v_2(x_r \cos\phi_r + y_r \sin\phi_r) < 0 \quad (21)$$

constitute the unsafe portion, and those $(x_r, y_r, \phi_r)$ on $\partial T$ for which

$$-v_1 x_r + v_2(x_r \cos\phi_r + y_r \sin\phi_r) = 0 \quad (22)$$

are the final state conditions for the boundary of the safe set $V_1$. To solve for $p(t)$ and $x(t)$ along this boundary for $t < t_f$, we must first determine $u^*(t_f)$ and $d^*(t_f)$. Equations (19) are not defined at $t = t_f$, since $s_1 = s_2 = 0$ on $\partial T$, giving rise to "abnormal extremals" (meaning that the optimal Hamiltonian loses dependence on $u$ and $d$ at these points). Analogously to [10] (Chapter 8), we use an indirect method to calculate $u^*(t_f)$ and $d^*(t_f)$: at any point $(x_r, y_r, \phi_r)$ on $\partial T$, the derivatives of the switching functions $s_1$ and $s_2$ are

$$\dot{s}_1 = y_r v_1 \quad (23)$$

$$\dot{s}_2 = x_r v_2 \sin\phi_r - y_r v_2 \cos\phi_r \quad (24)$$

For points $(x_r, y_r, \phi_r) \in \partial T$ such that $\phi_r \in (0, \pi)$ it is straightforward to show that $\dot{s}_1 > 0$ and $\dot{s}_2 > 0$, meaning that for values of $t$ slightly less than $t_f$, $s_1 < 0$ and $s_2 < 0$. Thus for this range of points along $\partial T$, $u^*(t_f) = -1$ and $d^*(t_f) = 1$. These values for $u^*$ and $d^*$ remain valid for $t < t_f$ as long as $s_1(t) < 0$ and



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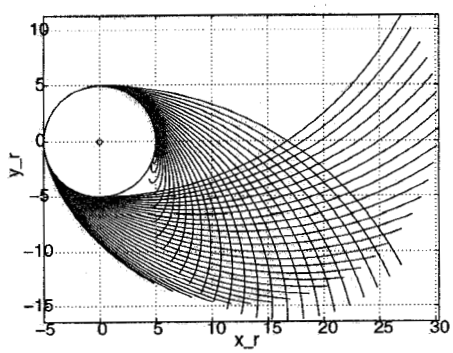



**Figure 5:** A top view of the unsafe set in the previous figure.

$s_2(t) < 0$. When $s_1(t) = 0$ and $s_2(t) = 0$, the saddle solution switches and the computation of the boundary continues with the new values of $u^*$ and $d^*$, thus introducing "kinks" into the safe set boundary. These points correspond to the shocks in the Hamilton-Jacobi equation discussed above. Figures 4 and 5 display the resulting boundary of the safe set $V_1$, for $t < t_f$ until the first time that either $s_1(t)$ or $s_2(t)$ switches. The automaton illustrating the *least restrictive control scheme* for safety is shown in Figure 6. The law is least restrictive in that the control $u$ is not restricted when the state is inside $V_1$. This automaton may be thought of as the composition of two switching automata, one each for the pursuer and evader. The individual switching automaton for $u$ is easily derived by neglecting transitions for $d$, and conversely. The computation of the boundary of $V_1$ is in general difficult and is a topic of our current research. For certain ranges of $\mathcal{U}$ and $\mathcal{D}$, the surfaces shown in Figure 4 intersect, and at the intersection it is not clear that $u^*$ is the unique safe input.

### 3.2 Resolution by Linear Velocity

We now consider the case in which the angular velocities of the two aircraft are zero, and collision is avoided by altering the velocity profile of the trajectories. Thus, $u = v_1$, $d = v_2$, and model (1) reduces to:

$$\begin{aligned} \dot{x}_r &= -u + d\cos\phi_r \\ \dot{y}_r &= d\sin\phi_r \\ \dot{\phi}_r &= 0 \end{aligned} \tag{25}$$

The input and disturbance lie in closed subsets of the positive real line $u \in \mathcal{U} = [\underline{v}_1, \overline{v}_1] \subset \mathbb{R}^+$, $d \in \mathcal{D} = [\underline{v}_2, \overline{v}_2] \subset \mathbb{R}^+$.

The Target set $T$ and function $l(x)$ are defined as in the previous example. In this example, it is straightforward to calculate the saddle solution $(u^*, d^*)$ directly, by integrating equations (25) for piecewise constant $u$ and $d$, and substituting the solutions into the cost function (4). To do this we first define the switching functions

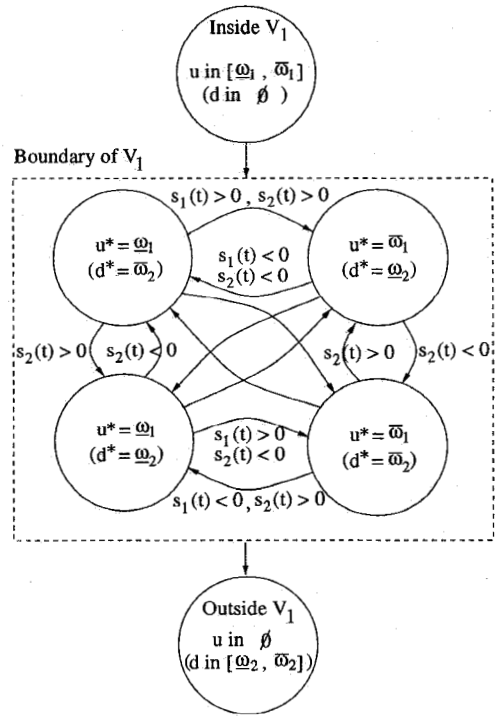



**Figure 6:** Switching law governing the two aircraft system with angular velocity control inputs.

$s_1$ and $s_2$ as

$$\begin{aligned} s_1(t) &= x_r \\ s_2(t) &= x_r\cos\phi_r + y_r\sin\phi_r \end{aligned} \tag{26}$$

**Proposition 1** [Saddle Solution for Linear Velocity Controls] *The global saddle solution $(u^*, d^*)$ to the game described by system (25) for the cost $J_1(x, t, u, d)$ given by equation (4) is*

$$u^* = \begin{cases} \underline{v}_1 & \text{if } sgn(s_1) > 0 \\ \overline{v}_1 & \text{if } sgn(s_1) < 0 \end{cases} \tag{27}$$

$$d^* = \begin{cases} \underline{v}_2 & \text{if } sgn(s_2) > 0 \\ \overline{v}_2 & \text{if } sgn(s_2) < 0 \end{cases} \tag{28}$$

**Proof:** In [13].

As can be seen from equation (27), the optimal speed of the evader depends on the position of the pursuer relative to the evader. If the pursuer is ahead of the evader in the relative axis frame, then $u^*$ is at its lower limit, if the pursuer is behind the evader in the relative axis frame then $u^*$ is at its upper limit. If the pursuer is heading towards the evader, then $d^*$ is at its upper limit, if the pursuer is heading away from the evader, $d^*$ is at its lower limit. The unsafe sets of states are illustrated in Figure 7 for various values of $\phi_r$, and speed ranges as illustrated.

## 4 Conclusions

In this paper, a conflict resolution strategy for free flight based Air Traffic Management is presented. A



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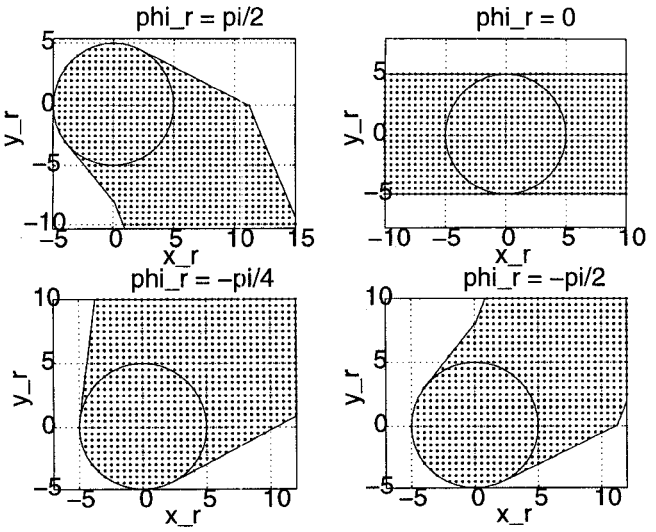



**Figure 7:** Unsafe sets $(x_r, y_r)$ for $[\underline{v}_1, \overline{v}_1] = [2, 4]$, $[\underline{v}_2, \overline{v}_2] = [1, 5]$ and $\phi_r = \pi/2$, 0, $-\pi/4$, and $-\pi/2$.

noncooperative method based on game theory is described, and two examples are presented to illustrate the methodology. In our current research, we are extending the verification methodology of this paper to dynamic rather than kinematic models of the aircraft, with different flight modes. In a hierarchical flight vehicle management system, such as the one proposed in [8], we have explained the role of dynamic models. Some preliminary results in this context have been presented in [15], [16].

## References


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