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# 2-1/2 D Conflict Resolution Maneuvers for ATMS[1]

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**Abstract**

This work is motivated by the current trend in the aviation community to move towards decentralized Air Traffic Management Systems (ATMS), in which the aircraft operate in "free flight" mode instead of following prespecified "freeways in the sky". Automatic conflict resolution strategies are an integral part of the free flight setting. Given the nature of the problem, the limited sensing capabilities, and the complex aircraft dynamics and constraints, it is very hard to solve the conflict resolution problem in its full generality. Indeed, the appropriate solution should be safe (collisions never occur), live (destinations are always reached), and flyable (dynamic constraints on accelerations and velocities are satisfied). This paper explores the use of distributed motion planning for multiple aircraft using vector field techniques. The coordination between aircraft is achieved using a series of horizontal and vertical planar avoidance maneuvers, resulting in 2-1/2 D solutions. The switching nature of the proposed solution makes it natural to model and analyze the resolution protocols using hybrid systems techniques.

## 1 Introduction and Motivation

Recent trends in air traffic control suggest the concept of "free flight" as a step towards a more efficient utilization of the airspace and the objectives of individual aircraft. Free flight distributes some of the control authority to the aircraft, thereby reducing the workload of air traffic controllers and making the system more reliable and less prone to the failures of the central controller. The conflict resolution strategies are at the top of the agenda for making free flight a reality. We consider the conflict resolution problem as a part of the overall ATMS architecture.

In the proposed ATMS architecture of [TPL+97], each aircraft follows a nominal path from source airport to destination airport described by a sequence of *waypoints*, which are fixed points in the airspace. This nominal path is calculated off-line in consultation with Air Traffic Control (ATC) and is designed to be optimal in some sense and conflict-free. However, bad weather, high winds, or schedule delays which cause conflicts with other aircraft may force the aircraft to deviate from this nominal route. In the current system, these deviations are calculated by the central ATC and each aircraft must obtain a clearance from ATC before altering its course. In the proposed ATMS, the aircraft may plan its own deviation trajectories without consulting ATC. This semi-autonomy is enabled by on-board conflict resolution algorithms.

In the conflict resolution algorithm, we would like to guarantee: *safety, liveness, and the generation of flyable maneuvers.* While the first two notions are clearly defined in hybrid system theory, the third is related to dynamical considerations. In air traffic conflict resolution, liveness means that regardless of the initial condition, one can generate trajectories which will always achieve the desired goal; safety guarantees that collisions never occur; and flyability conditions guarantee that the trajectories will obey dynamic constraints expressed in terms of limits on accelerations and velocities. Given the limited sensing capabilities of each aircraft and the dynamic constraints, it is very hard to solve the conflict resolution problem in its full generality. While there have been many techniques proposed in the past, each of them fails short of one of the above requirements.

This work extends [KTPS97], in which we construct various parameter-dependent maneuvers for two, three, and four aircraft using distributed motion planning/vector field techniques for coordination in the horizontal plane of multiple aircraft. The appeal of the potential field techniques is partly the computational efficiency and partly the flexibility of adjusting individual parameters of the controllers in order to represent the behavior of each aircraft. The vector field parameters capture some of the qualitative nature of the maneuvers and can also model the level of cooperation between the aircraft involved in the conflict resolution. However, due to the distributed nature of the algorithm it is difficult to provide flyability guarantees without trading off safety.

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In the current paper, we extend the maneuvers to "2-1/2" dimensions, meaning that the aircraft can also change altitude to avoid conflict. This generalization adds an extra dimension to the conflict resolution maneuvers, and allows for the resolution of conflicts which are caused by aircraft involved in altitude changes. This additional degree of freedom reduces the probability of unflyable maneuvers and allows the resolution of conflicts without changing the airspeed of the aircraft thus preserving the limit on lateral acceleration. This contribution attempts to partially automate the process of trajectory generation in cases where the trajectories generated using potential fields in the horizontal plane only are not suitable, while keeping in mind the preferences observed in current air traffic control.

## 2 Related Work

The hybrid nature of the conflict resolution problem for multiple aircraft comes from bringing together two main aspect of the problem: the motion (trajectory generation) aspect and decision making/coordination aspect.

There are many studies in the classical motion planning literature dealing with the multiple robot planning problem. Algorithms embedded in time-extended configuration space [ELP87] prove to be computationally hard, and with additional velocity bounds even the single-agent motion planning problem is NP-complete [DXCJ93]. In robotics applications the scenarios considered most often involve navigation in the presence of other moving agents and obstacles [Mat95]. The proposed solutions are geared towards a distributed setting, in which only the local information about the state of the environment and the other agents in the vicinity is available to each agent. An attempt to guarantee in such a distributed setting that the agents achieve their goals without colliding with each other has been proposed by [Mas96]. These distributed approaches are typically formulated using vector field methods. In [FK90], kinematic constraints are incorporated by using vector field methods in the velocity space of a single agent. Additional appeal of these methods in the simplest dynamic settings is the ease of obtaining the actual control laws for reaching the goal. For the single-agent setting, [RK93] proposes a class of navigation functions for which the convergence properties and stability of control laws are guaranteed globally.

In spite of the fact that collision avoidance is a necessary part of agents' navigational capabilities, the requirements for safety and optimality have not been addressed to any great extent. This is partly due to the fact that the agent velocities have traditionally been relatively small and safety issues not so prominent: low velocity collisions occasionally occur, but various recovery strategies allow the agents to further pursue their tasks. In path planning for more than two agents, prioritized schemes have been used to fix the order in which the conflicts are resolved. In [CI93] it was shown that for air traffic maneuvers which are constrained by the turning angle and the maximum deviation from the planned trajectory, certain priority orderings failed to resolve the conflict.

Current research endeavors in conflict prediction and resolution for air traffic systems include [YK97, PE97, KMH96, ZS97, OF97, TPS98]. Conflict prediction could be spatial, temporal or probabilistic. Spatial and temporal approaches, such as [KMH96, OF97], calculate the four dimensional coordinates of a possible conflict. Probabilistic approaches, such as [YK97, PE97], assume stochastic uncertainty in the measured information and determine the probability of collision. The work of [KMH96, ZS97] formulates conflict resolution as an optimal control problem whereas [OF97] treats the problem as a convex optimization problem. In [TPS98], the emphasis is on proving that the maneuvers are safe once generated: in the current paper, we study the generation of these maneuver using potential and vortex fields.

## 3 Maneuver Generation

In this paper we assume that there are no stationary obstacles, and we consider each aircraft to be a "hockey puck" of a specified radius and height (2.5 nautical miles, 2000 ft) representing the desired clearance from the other aircraft.

In order to accommodate maneuvers in both the vertical and horizontal planes, we consider vector fields in 3-D space and compute the actual control commands by projecting the force vector into the plane in which the maneuver is being carried out. The planner is obtained by the superposition of several vector fields representing qualitatively different steering actions of each agent. Suppose we have $m$ agents, with the $i$th agent represented by a cylinder with radius $r_i$ and its configuration denoted by $\boldsymbol{x}_i = (x_i, y_i, z_i)$. The desired destination of the $i$th agent $\boldsymbol{x}_{di} = (x_{di}, y_{di}, z_{di})$ is encoded by an attractive potential function:

$$U_a(\boldsymbol{x}_i, \boldsymbol{x}_{di}) = \frac{1}{2}(\boldsymbol{x}_{di} - \boldsymbol{x}_i)^2$$

In order to achieve the desired destination, a force proportional to the negative gradient of $U_a$ needs to be exerted on the $i$th aircraft:

$$F_a(\boldsymbol{x}_i, \boldsymbol{x}_{di}) = -\nabla U_a(\boldsymbol{x}_i, \boldsymbol{x}_{di}) = -(\boldsymbol{x}_i - \boldsymbol{x}_{di})$$

To prevent collisions between agents $i$ and $j$, the following spherically symmetric repulsive field $U_r(\boldsymbol{x}_i, \boldsymbol{x}_j)$



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is associated with each agent:

$$U_r(\boldsymbol{x}_i, \boldsymbol{x}_j) = \begin{cases} -\frac{(\boldsymbol{r}_{ij} - (\boldsymbol{r}_j + \delta_{rj}))^2}{2\delta_{rj}} & \text{if } \boldsymbol{r}_j \leq \boldsymbol{r}_{ij} \leq \boldsymbol{r}_j + \delta_{rj} \\ 0 & \text{otherwise} \end{cases}$$

where $\boldsymbol{r}_{ij} = \|\boldsymbol{x}_i - \boldsymbol{x}_j\|$ is the distance between the centers of the $i$th and $j$th agents, $\boldsymbol{r}_j$ is the radius of $j$th agent, and $\delta_{rj}$ is the influence zone of its repulsive field. The repulsive force associated with this field is:

$$F_r(\boldsymbol{x}_i, \boldsymbol{x}_j) = \nabla U_r(\boldsymbol{x}_i, \boldsymbol{x}_j)$$

Since both the attractive and repulsive vector fields are in 3-D, the field obtained by their superposition is also in 3-D. Therefore, the potential field construction may force the aircraft to perform a very demanding resolution maneuver. Motivated by current air traffic control practice, we would like aircraft to perform either planar (horizontal) or altitude maneuvers. Horizontal maneuvers will be given a higher priority since they are preferable by air traffic controllers who visualize radar information on 2-D displays. Therefore, we *project* the 3-D vector field of each agent onto the horizontal and vertical planes (defined by the body frame of the aircraft which are assumed to be cruising at level flight and constant heading). The horizontal projection is then compared to the vertical one. If the horizontal projection is larger, the aircraft in question will perform a horizontal maneuver. If the vertical projection is larger, the aircraft performs an altitude maneuver.

For horizontal maneuvers, another useful component for conflict resolution is the vortex field component, which mediates the coordination between individual agents. Since a smooth vortex field on a sphere must have at least one equilibrium, the extension of the vortex field technique from 2-D to 3-D is not straightforward. For this reason we define the vortex field as a 2-D component which acts in the plane in which the maneuver is performed. The 2-D vortex field, used to ensure that all agents turn in the same direction when encountering a conflict, is constructed around each agent tangentially to the projected repulsive field $U_r(\boldsymbol{x}_i, \boldsymbol{x}_j)$:

$$F_p(\boldsymbol{x}_i, \boldsymbol{x}_j) = \Pi_\perp(F_r(\boldsymbol{x}_i, \boldsymbol{x}_j))$$

where $\Pi_\perp : \Re^3 \rightarrow \Re^2$ projects the vector to the plane in which the maneuver is taking place. Denoting the components of the projected vector in that plane as $[u, v]^T$, the vortex field associated with the corresponding repulsive field is $F_v(\boldsymbol{x}_i, \boldsymbol{x}_j) = [-v, u]^T$ or $F_v(\boldsymbol{x}_i, \boldsymbol{x}_j) = [v, -u]^T$ which corresponds to counterclockwise and clockwise vortex flow respectively. Setting the direction to a particular sign for all agents corresponds essentially to a "rule of the road" which specifies the direction of the avoidance for conflict maneuvers.

The dynamic planner for a single agent in the presence of multiple agents is obtained by superposition of participating attractive, repulsive, and vortex forces:

$$\dot{\boldsymbol{x}}_i = \frac{F_a(\boldsymbol{x}_i, \boldsymbol{x}_{di})}{\|F_a(\boldsymbol{x}_i, \boldsymbol{x}_{di})\|} + \sum_j (k_{ri} F_p(\boldsymbol{x}_i, \boldsymbol{x}_j) + k_{vi} F_v(\boldsymbol{x}_i, \boldsymbol{x}_j))$$

where $j = 1, ..., m,\ i \neq j$. The contributions from repulsive and vortex forces range between [0, 1], increasing as the agent approaches the boundary of another agent. Normalization of the attractive field component makes its contribution comparable to the magnitudes of the repulsive and vortex fields. The strength of the field then becomes independent of the distance to the goal, capturing merely the heading to the goal. The individual contributions are then weighted by $k_{ri}$, $k_{vi}$ and the resulting vector is again normalized and scaled by $k_{di}$, a constant proportional to the desired velocity of the $i$th agent. The velocity of $i$th agent is then:

$$\boldsymbol{v}_i = k_{di} \frac{\dot{\boldsymbol{x}}_i}{\|\dot{\boldsymbol{x}}_i\|}$$

In Figure 1, two agents having different velocities participate in an overtake maneuver; agent 1 is 1.5 times faster than agent 0. In the top maneuver, agent 1 overtakes agent 0 and agent 0 moves away from agent 1, resulting in small deviations for both agents from their original trajectories. The strength of the contribution from the repulsive and vortex fields is the same for both agents. Willingness of the slower agent to cooperate in the overtake maneuver can be modeled by the strength of the agent's repulsive and vortex fields: in the bottom maneuver of Figure 1 the contributions of agent 2's vortex and repulsive fields are set to zero and agent 2 does not deviate from its original trajectory. In conflicts involving only two aircraft the number of possible conflict scenarios is quite low. When multiple ($\geq 3$) aircraft are involved the vector field based planner is very

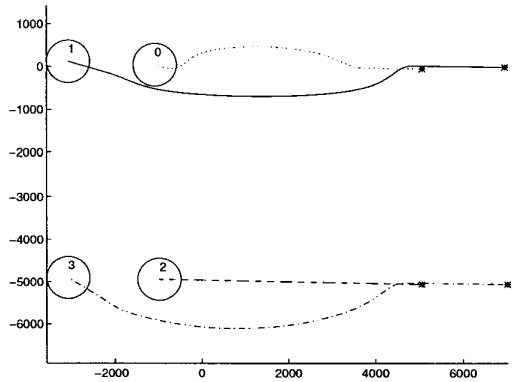



**Figure 1:** *Overtake* maneuvers. Top: $1.5k_{d0} = k_{d1}$, $k_{r0} = k_{r1} = k_{v0} = k_{v1} = 1.0$. Bottom: $1.5k_{d2} = k_{d3}$, $k_{r2} = k_{v2} = 0.0; k_{r3} = k_{v3} = 1.0$.



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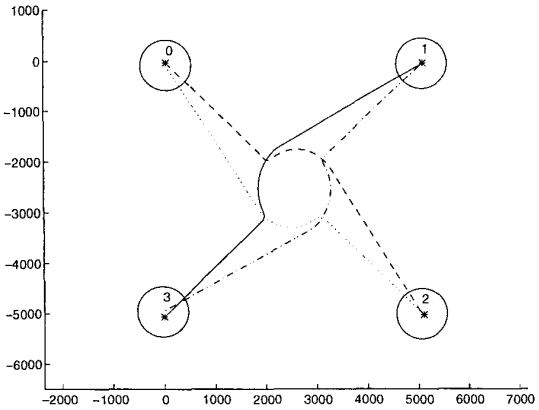



**Figure 2:** Symmetric *roundabout*, gain factors for individual agents are the same.

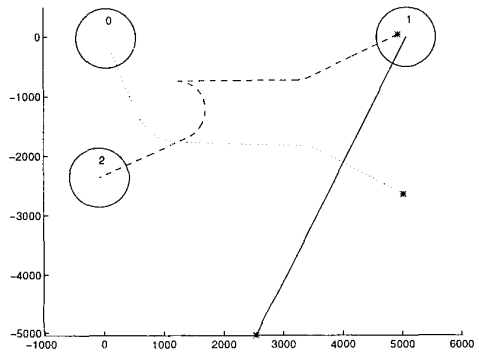



**Figure 3:** General conflict scenario. Trajectory of agent 2 is not flyable.

instructional: the direction of the vortex field contribution serves as a coordination element between the aircraft. Figure 2 depicts a symmetric *roundabout* maneuver similar to the one proposed in [TPS98]. The agents involved in the resolution of the conflict are homogeneous, having the same velocities, participating equally in the maneuver. However a general multi-aircraft conflict as the one in Figure 3 can often result in an unflyable maneuver, in this case for aircraft 2.

In order to address the flyability issues we first characterize what it means for a trajectory to be flyable and how to possibly guarantee the dynamic constraints.

**Flyable trajectories:** One of the specifications of any conflict resolution algorithm is to produce *flyable* resolution trajectories: the airspeed of aircraft $i$ is always positive and bounded away from zero (due to stall limits $V^i_{min}$), and hard limits on the angular acceleration are respected. Due to the distributed nature of the planner, we cannot guarantee that the trajectories will be flyable. Even though the potential field equations do not have any orientation information, we can use the notion of differential flatness in order to obtain it. Consider, for example, planar resolution trajectories where the potential field generates a trajectory $(x_i(t), y_i(t))$ for aircraft $i$. The orientation $\theta_i$ of aircraft $i$ can then be obtained as the solution of the following equation

$$\dot{x}_i \sin\theta_i - \dot{y}_i \cos\theta_i = 0 \tag{1}$$

By differentiating the above equation we obtain

$$(\dot{x}_i \cos\theta_i + \dot{y}_i \sin\theta_i)\dot{\theta}_i = \ddot{y}_i \cos\theta_i - \ddot{x}_i \sin\theta_i \tag{2}$$

From equation (2), the flyability constraints are now clear.

- **Positive Speed above Stall Limit**

$$\dot{x}_i \cos\theta_i + \dot{y}_i \sin\theta_i \geq V^i_{min} \tag{3}$$

- **Turning Radius Constraint**

$$\dot{\theta}_i = \frac{\ddot{y}_i \cos\theta_i - \ddot{x}_i \sin\theta_i}{\dot{x}_i \cos\theta_i + \dot{y}_i \sin\theta_i} \in [-\Omega_i, \Omega_i] \tag{4}$$

where $\Omega_i$ represents the angular acceleration limit. Similar conditions can be obtained for 3-D trajectories.

Figure 4 presents an example where conflict resolution resulted in an unflyable maneuver. In spite of the fact that currently we cannot guarantee that the maneuver will be flyable, following equation (2) we can detect that the trajectory is unflyable and adjust the maneuver accordingly.

**Algorithm:** The algorithm for generating the trajectories and adjusting them in case they are unflyable takes into account the fact that horizontal maneuvers are preferable over altitude maneuvers because of passenger comfort considerations. The maneuver preference in our algorithm is as follows:

1. while flying at a particular altitude first consider 2-D maneuvers (in the horizontal plane);
2. if unflyable trajectories result, force some of the aircraft to resolve the conflict by making altitude changes and returning to the original altitude;

We demonstrate how to incorporate preferences outlined in the algorithm and adjust the unflyable maneuvers without changing the speed of the aircraft. In Figure 4 the trajectory of aircraft 1 is not flyable due to the cusp in the middle section of the trajectory. By directing aircraft 1 (Figure 5a,b) to resolve the conflict by performing an altitude change maneuver, all of the



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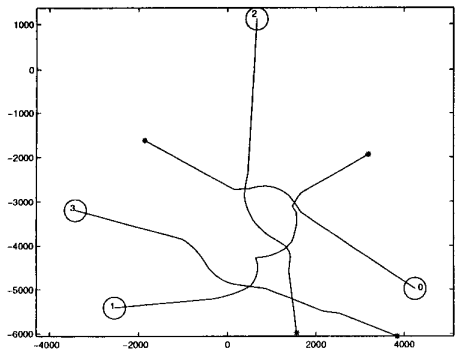
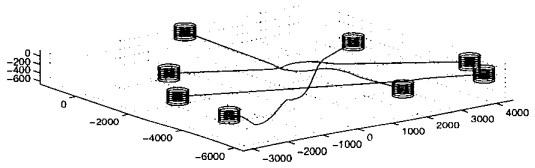


**Figure 4:** Original asymmetric maneuver is unflyable for aircraft 1.



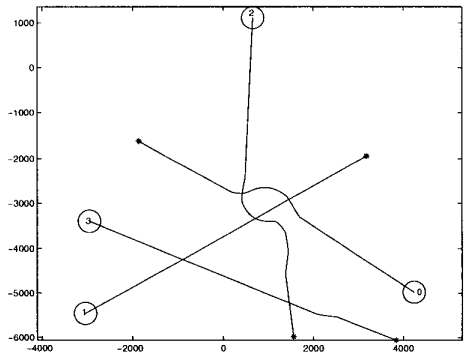


**Figure 5:** a) Aircraft 1 is directed to resolve the maneuver making an altitude change. b) Projection of the resulting maneuver onto the horizontal plane.

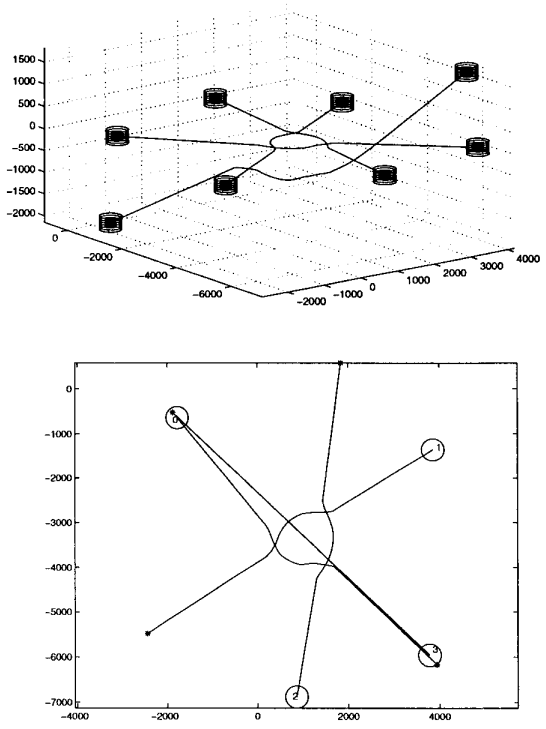


**Figure 6:** a) Aircraft 3 is making an altitude change maneuver while passing through an altitude with heavy traffic where the aircraft participate in a *roundabout* maneuver. b) Horizontal projection of the resulting maneuver.

resulting trajectories are flyable and the deviations of other agents are more graceful.

Another example of a conflict which can be resolved by extending the vector field technique to 3-D is in Figure 6. In this example, aircraft 3 is moving from lower to higher altitude passing through a level with heavy traffic. While all the aircraft in the middle level pursue their original trajectories and resolve the conflicts with each other, aircraft 3 is repelled by them and is forced to level out and then resume the altitude change.

## 4 Issues for further research

In this paper, we have presented an outline of a conflict resolution algorithm for $n$ aircraft in $2-1/2$ dimensions. The algorithm considers safety, and the generation of flyable trajectories. The distributed nature of the algorithm as well as the multiple desired objectives naturally give rise to switching policies as demonstrated by the classification of aircraft performing horizontal and vertical maneuvers.



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