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Opportunistic sensor scheduling in wireless control systems

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***Abstract*— We consider a wireless control system where multiple power-constrained sensors transmit plant output measurements to a controller over a shared wireless medium. A centralized scheduler, situated at the controller, grants channel access to a single sensor on each time step. Given plant and controller dynamics, we design scheduling and transmit power policies that adapt opportunistically to the random wireless channel conditions experienced by the sensors. The objective is to obtain a stable system, by minimizing the expected decrease rate of a given Lyapunov function, while respecting the sensors' power constraints. We develop an online optimization algorithm based on the random channel sequence observed during execution which converges almost surely to the optimal protocol design.**

I. INTRODUCTION

Wireless control systems in, e.g., industrial or building automation applications, often involve sensing and actuating devices at different physical locations that communicate control-relevant information over shared wireless mediums. Scheduling access to the medium is critical to avoid interferences between transmissions but also affects the overall control performance. Previous work in wired and/or wireless networked control systems, focused on deriving stability conditions under given scheduling protocols – see, e.g., [1]–[3]. Stability is typically examined by a switching system reformulation [2], often under additional network phenomena such as communication delays, uncertain communication times, packet drops.

Beyond the question of stability, the problem of designing schedulers suitable for control applications has also been addressed. The proposed protocols can be generally classified as either fixed or dynamic. Typical examples of the first type are periodic protocols, i.e., repeating in a predefined sequence (e.g., round-robin). Fixed protocols leading to stability [4], controllability and observability [5], or minimizing linear quadratic objectives [6] have been proposed. Deriving otherwise optimal scheduling sequences is recognized as a hard combinatorial problem [7]. Dynamic scheduler design constitutes a different approach based on the current plant/control system states, where, informally, the subsystem with the largest state discrepancy is scheduled to communicate – see examples in, e.g., [2], [8]–[10].

In this paper we focus on designing scheduling protocols for wireless control systems and, in contrast to the above approaches, we examine how to opportunistically exploit the channel conditions affecting the transmissions on the shared wireless medium. Channel conditions refer to the channel fading effects which vary randomly over time and also differ among the users [11, Ch. 14]. As a result, dynamically assigning channel access based on current channel conditions can exploit the fact that at different points in time transmissions for some users become more favorable than others. Such channel-aware mechanisms have been developed for wireless networking frameworks [12]. In previous work we have shown how they can be adopted when scheduling independent control tasks whose control performance requirements translate to different channel utilization demands [13]. Here we consider the problem of scheduling different sensors transmitting outputs of a single plant to a controller when the sensors have limited power resources (Section II). These power resources can be used to counteract channel fading effects during transmission and obtain a higher decoding probability at the receiver/controller [14]. Sensor scheduling should make an efficient use of the available power resources, while additionally it should lead to a closed loop control system with stability guarantees.

We formulate the design of channel-aware scheduling and power allocation policies in a stochastic optimization framework (Section II-A), under the sensors' power constraints. The objective is to optimize a closed-loop stability margin measured as the decrease rate of a given Lyapunov function, in expectation over the random channel conditions. Conceptually, the Lyapunov function helps to abstract control performance in a single-time-step, avoiding the complexity of designing scheduling sequences over time horizons as in other approaches, e.g., [7]. Based on the Lagrange dual of our problem, an optimization algorithm is developed in Section III. The algorithm does not require prior knowledge of the channel distribution, but can be implemented instead as an online protocol based on a random observed channel sequence. We show that the protocol converges almost surely to the tightest stability margin and meets the power constraints. We note that in general this does not imply system stability. However, if the system is stabilizable with respect to the selected Lyapunov function, the online protocol leads to a stable system. We close with numerical simulations and conclusions.

Notation: A set of variables $a_0, a_1, \dots a_k$ is denoted compactly as $a_{0:k}$. We denote by $\geq, \succeq, \succ$ the comparison with respect to the cones of $\mathbb{R}_+^m$, of the real $n \times n$ symmetric positive semi-definite matrices S_+^n, and of the real $n \times n$ symmetric positive definite matrices S_{++}^n respectively.

II. PROBLEM FORMULATION

We consider the wireless control architecture of Fig. 1 where m sensors measuring plant outputs communicate over

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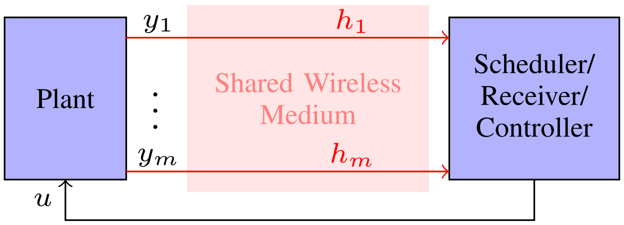

Fig. 1. Opportunistic sensor scheduling in a wireless control architecture. Each sensor i measures and transmits a plant output y_i to a centralized controller over a shared wireless medium. A scheduler at the receiver/controller opportunistically selects which sensor transmits at each time step based on the random channel conditions $h_1, \ldots, h_m$ experienced by the sensors.

a shared wireless medium to a controller. A centralized scheduler selects at most one sensor to access the medium at each time step. Due to uncertainties in the wireless channel, which we model in detail next, the transmitted sensor measurements might get lost.

Let $\gamma_{i,k} \in \{0,1\}$ denote the event that sensor i is scheduled at the discrete time step k *and* the respective transmission is successful. Let also $\gamma_{0,k} \in \{0,1\}$ denote the event that no sensor transmits successfully at time k. Let $x_k \in \mathbb{R}^n$ denote the overall state of the system before transmission at time k. System evolution from x_k to x_{k+1} depends on whether a successful transmission occurs at time k and which sensor transmits. Suppose the system follows linear dynamics denoted by $A_i \in \mathbb{R}^{n\times n}$ if sensor i transmits successfully ($\gamma_{i,k} = 1$), and $A_0 \in \mathbb{R}^{n\times n}$ when no sensor transmits ($\gamma_{0,k} = 1$). We describe then the system evolution by the switched linear discrete time system

$$x_{k+1} = \sum_{i=0}^{m} \gamma_{i,k}\, A_i x_k + w_k. \tag{1}$$

with w_k modeling an independent identically distributed (i.i.d.) noise process with mean zero and covariance $W \succeq 0$. We note that the state x_k may contain in general not only the plant state, but also the controller state if the controller is dynamic – see [1], [2] for examples of model (1).

Given the overall system dynamics, we focus on designing the wireless communication (scheduling and associated transmit power) which affects the transmission indicators $\gamma_{i,k}$. We describe the wireless channel conditions for link i, between sensor i and the controller, at time k by the channel fading coefficient $h_{i,k}$ that sensor i experiences if it transmits at time k. Due to propagation effects, the channel fading states $h_{i,k}$ change unpredictably [11, Ch. 3] and take values in a subset $\mathcal{H} \subseteq \mathbb{R}_+$ of the positive reals. Channel states $h_{i,k}$ change not only over time k but also between sensors i. We group $h_{i,k}$ for $1 \leq i \leq m$ at time k in a vector $h_k \in \mathcal{H}^m$, and we adopt a block fading model whereby h_k are random variables independent across time slots k and identically distributed with a multivariate distribution ϕ on $\mathcal{H}^m$. Channel states are also independent of the plant process noise w_k. We make the following technical assumption to avoid a degenerate channel distribution, but otherwise no other prior information about the channel distribution will be needed for the communication design in this paper.

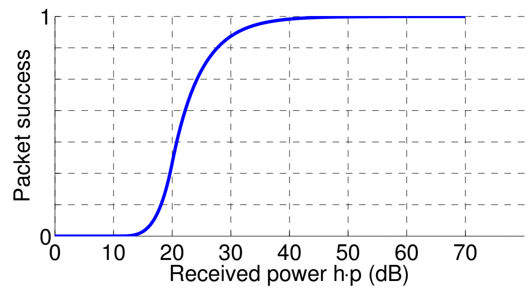

Fig. 2. Probability of successful decoding $q(hp)$ for practical forward error-correcting codes. The probability of successful decoding is a sigmoid function of the received SNR $\sim h\,p$.

Assumption 1. The joint distribution ϕ of channel states h_k has a probability density function on $\mathcal{H}^m$.

If sensor i is scheduled to transmit at time k it selects a transmit power level $p_{i,k} \in [0, p_{\max}]$. Channel fading and transmit power affect the probability of successful decoding of the message at the receiver. In particular, given the forward error-correcting code in use, the probability q that a packet is successfully decoded is a function of the received signal-to-noise ratio (SNR). The SNR is proportional to the received power level expressed by the product $h\,p$ of the channel fading state and the allocated transmit power. Overall we express the probability of success by some given relationship of the form $q(h_{i,k}, p_{i,k})$ – see [14] for more details on this model. An illustration of this relationship is shown in Fig. 2. The assumptions on the function $q(hp)$ are the following.

Assumption 2. The function $q(.)$ as a function of the product $r = h\,p$ for $r \geq 0$ satisfies:

(a) $q(0) = 0$,
(b) $q(r)$ is continuous, and strictly increasing when $q(r) > 0$, i.e., for any $r' > r$ it holds that $q(r') > q(r)$,
(c) for any $\mu \geq 0$ and for almost all values $h \in \mathcal{H}$ the set $\operatorname{argmin}_{0 \leq p \leq p_{\max}} p - \mu q(h\,p)$ is a singleton.

Parts (a),(b) of this assumption state that the probability of successful decoding $q(h\,p)$ will be zero when the received power level $h\,p$ is small, and it becomes positive $q(h\,p) > 0$ and strictly increasing for larger values of $h\,p$. Part (c) is a more stringent assumption introduced for technical reasons as we explain in the sequel. As shown in Fig. 2 for cases of practical interest $q(h\,p)$ has a sigmoid form and satisfies all the above requirements.

Before transmission, a scheduler selects which sensor will access the channel. We allow for randomized scheduling and we denote with $\alpha_{i,k}$ the probability that sensor i is selected at time k. For simplicity we require that exactly one sensor is scheduled, meaning that $\sum_{i=1}^{m} \alpha_{i,k} = 1$. Hence the scheduling decision $\alpha_{i,k}$ for $i = 1, \ldots, m$ can be grouped as a vector α_k selected from the probability simplex

$$\alpha_k \in \Delta_m = \Big\{ \alpha \in \mathbb{R}^m : \alpha \geq 0, \sum_{i=1}^{m} \alpha_i = 1 \Big\}, \tag{2}$$

Given scheduling $\alpha_k \in \Delta_m$, power allocation $p_k \in [0, p_{\max}]^m$, and channel state $h_k \in \mathcal{H}^m$, we model the transmission events $\gamma_{i,k}$ as Bernoulli random variables with

$$\mathbb{P}[\gamma_{i,k} = 1 \,\big|\, h_k, \alpha_k, p_k] = \alpha_{i,k}\, q(h_{i,k}, p_{i,k}) \tag{3}$$

This states that the probability that sensor i successfully transmits equals the probability that i is scheduled to transmit *and* the message is correctly decoded at the receiver.

Our goal is to design scheduling and power allocation protocols that adapt to the channel conditions on the shared wireless medium in order to make an efficient use of the sensors' power resources and lead to a stable control system. The exact problem specification is presented next, after a remark on the practical implementation of the protocol.

Remark 1. The proposed scheduler of the architecture in Fig. 1 is assumed to have information about the current channel conditions on the shared medium. Channel conditions on each wireless link can be measured by pilot signals sent from the sensors to the receiver/controller at each time step before the scheduling decision. Depending on the measured conditions, the scheduler at the receiver selects and notifies via the reverse channel a sensor to transmit. Channel state information can also be passed this way back to the selected sensor, which accordingly adapts its transmit power. □

A. Communication design specification

We consider scheduling and power variables α_k, p_k that adapt to the current channel states h_k, so they can be expressed as mappings $\alpha_k = \alpha(h_k)$, $p_k = p(h_k)$ of the form

$$\mathcal{A} = \{\alpha : \mathcal{H}^m \to \Delta_m\},\ \mathcal{P} = \{p : \mathcal{H}^m \to [0, p_{\max}]^m\}. \tag{4}$$

Since channel states h_k are i.i.d. over time k these mappings do not need to change over time. Substituting $\alpha(.), p(.)$ in our communication model (3) and taking the expectation with respect to the channel state $h_k \sim \phi$, the expected probability of successful transmission for a sensor i at time k becomes

$$\begin{aligned}\mathbb{P}(\gamma_{i,k} = 1) &= \mathbb{E}_{h_k} \left\{\mathbb{P}[\gamma_{i,k} = 1 \,\big|\, h_k, \alpha(h_k), p(h_k)]\right\} \\ &= \mathbb{E}_h \alpha_i(h)\, q(h_i, p_i(h)).\end{aligned} \tag{5}$$

In the last equality we dropped the index of the channel variable h_k since they are i.i.d. with distribution ϕ over time k. This implies that the probabilities in (5) become constant for all k. Similarly the event that no sensor transmits ($\gamma_{0,k} = 1$) happens with a constant probability

$$\mathbb{P}(\gamma_{0,k} = 1) = 1 - \sum_{i=1}^{m} \mathbb{P}(\gamma_{i,k} = 1), \tag{6}$$

since the events on the right hand side are disjoint.

The goal of the communication design is to make an efficient use of the power resources available at the sensors while ensuring that the resulting control system is stable. In particular suppose each sensor i has a power budget b_i and we require that the expected power consumption induced by the communication design at each slot k is limited to

$$\mathbb{E}_h \alpha_i(h) p_i(h) \le b_i, \quad \text{for all } i = 1, \ldots, m. \tag{7}$$

The expectation on the left hand side is with respect to the channel distribution $h_k \sim \phi$ and accounts for the consumed transmit power whenever sensor i is scheduled.

Next we motivate the control system stability specification. Under the described communication design the transmission sequence $\{\gamma_{i,k}, 0 \le i \le m,\ k \ge 0\}$ is independent of the system state x_k. The resulting system (1) becomes a random jump linear system with i.i.d. jumps since the probabilities $\mathbb{P}(\gamma_{i,k} = 1)$ for $i = 0, 1, \ldots, m$ are constant over time k. Necessary and sufficient stability conditions for such systems are known. In particular, [15, Cor. 1] states that the system is mean square stable, i.e., the limits $\lim_{k\to\infty} \mathbb{E}x_k$ and $\lim_{k\to\infty} \mathbb{E}x_k x_k^T$ exist and are finite, if and only if there exists a matrix $P \in S_{++}^n$ satisfying

$$\sum_{i=0}^{m} \mathbb{P}(\gamma_{i,k} = 1) A_i^T P A_i \prec P. \tag{8}$$

The intuition behind (8) is that for fixed probabilities $\mathbb{P}(\gamma_{i,k} = 1)$ a Lyapunov-like function $V(x) = x^T P x$, $x \in \mathbb{R}^n$ decreases in expectation at each step. In particular, (8) is equivalent to

$$\begin{aligned}\mathbb{E}\left[V(x_{k+1}) \,\big|\, x_k\right] &= \sum_{i=0}^{m} \mathbb{P}(\gamma_{i,k} = 1) x_k^T A_i^T P A_i x_k + Tr(PW) \\ &< V(x_k) + Tr(PW)\end{aligned} \tag{9}$$

holding for any $x_k \in \mathbb{R}^n$, where the first equality follows from (1). Motivated by this observation about stability, we pose the problem of designing wireless communication variables that make the decrease rate in (9) as low as possible.

Suppose a quadratic Lyapunov function $V(x) = x^T P x$, $x \in \mathbb{R}^n$, with $P \in S_{++}^n$, is fixed. We are interested in channel-aware scheduling and power allocation variables (cf. (4)) that minimize the expected decrease rate of $V(x)$ (cf. (9)) and also meet the power budgets (7). We pose this as a stochastic optimization problem of the form

Optimal scheduling and power allocation design

$$\underset{r,\, \alpha\in\mathcal{A},\, p\in\mathcal{P}}{\textbf{minimize}} \quad r^2 \tag{10}$$

$$\textbf{subject to} \quad \mathbb{E}_h \alpha_i(h) p_i(h) \le b_i, \quad i = 1, \ldots, m \tag{11}$$

$$D_0 - \sum_{i=1}^{m} \mathbb{E}_h \alpha_i(h)\, q(h_i, p_i(h)) D_i \preceq rP \tag{12}$$

where for compactness we defined

$$D_0 = A_0^T P A_0, \quad D_i = A_0^T P A_0 - A_i^T P A_i, \tag{13}$$

for $i = 1, \ldots, m$. The semidefinite constraint (12) follows from (8) by substituting the probabilities (5), (6) induced from the communication design and by introducing an auxiliary variable r for the Lyapunov decrease rate (or increase if $r > 1$). The objective in (10) is an increasing function of r, so that the optimal rate is as small as possible, and for convenience is chosen to be strictly convex. For technical reasons we keep an implicit constraint $0 \le r \le r_{\max}$, which is not restrictive. The left hand side in (12) will always be bounded since the terms in expectations are probabilities (bounded by 1). Finally we note that problem (10)-(12) is always strictly feasible. Consider for instance the case $p \equiv 0$, implying a strict inequality in (11), and $r \ge 0$ sufficiently large so that (12) holds with strict inequality as well.

We denote the optimal value of the problem by P^* and an optimal solution by $r^*, \alpha^*(.), p^*(.)$. Even though the problem is infinite-dimensional (the variables $\alpha(.), p(.)$ are functions) and non-convex in general, in the following section we present an algorithm based on the Lagrange dual problem which converges to the optimal solution. Moreover the algorithm does not require any prior knowledge of the channel distribution, but can be implemented using the channel states measured online during execution.

III. Optimal scheduling and power allocation

In this section we present an algorithm that converges to the optimal channel-aware scheduling and power allocation policy. The algorithm employs the Lagrange dual problem of (10)-(12) and exploits the fact that there is no duality gap. Moreover, the algorithm can be implemented online based on a random channel sequence and converges almost surely to the optimal operating point with respect to (10)-(12).

To define the Lagrange dual problem of (10)-(12) consider dual variables $\nu_i \geq 0$ corresponding to each of the $i = 1, \ldots, m$ power constraints in (11), grouped in a vector $\nu \in \mathbb{R}^m_+$, and a symmetric positive semidefinite matrix $\Lambda \in \mathbb{S}^n_+$ corresponding to the semidefinite constraint (12). The Lagrangian is written as

$$L(r, \alpha, p, \nu, \Lambda) = r^2 + \sum_{i=1}^m \nu_i [\mathbb{E}_h \alpha_i(h) p_i(h) - b_i] + Tr(\Lambda[D_0 - \sum_{i=1}^m \mathbb{E}_h \alpha_i(h)\, q(h_i, p_i(h)) D_i - rP]), \quad (14)$$

while the dual function is defined as

$$g(\nu, \Lambda) = \min_{r,\, \alpha \in \mathcal{A},\, p \in \mathcal{P}} L(r, \alpha, p, \nu, \Lambda). \quad (15)$$

We will refer to any solution triplet r, α, p that minimizes the Lagrangian at a dual point ν, Λ by $r(\nu, \Lambda), \alpha(\nu, \Lambda), p(\nu, \Lambda)$. We will also denote by $\alpha(\nu, \Lambda; h), p(\nu, \Lambda; h)$ the value of these functions at a point $h \in \mathcal{H}^m$. We define then the Lagrange dual problem as

$$D^* = \underset{\nu \in \mathbb{R}^m_+, \Lambda \in \mathbb{S}^n_+}{\text{maximize}} \quad g(\nu, \Lambda). \quad (16)$$

By standard Lagrange duality theory the dual function $g(\nu, \Lambda)$ at any point ν, Λ is a lower bound on the optimal cost P^* of problem (10)-(12), hence also $D^* \leq P^*$ (weak duality). The following proposition however establishes a strong duality result ($D^* = P^*$). This is based on the results about similar stochastic optimization problems [12] and is a consequence of the assumption that channel distributions are absolutely continuous. A relationship between the optimal primal and dual variables is also provided.[1]

Proposition 1. *Let Assumption 1 hold, let P^* be the optimal value of the optimization problem (10)-(12) and (r^*, α^*, p^*) be an optimal solution, and let D^* be the optimal value of the dual problem (16) and ν^*, Λ^* be an optimal solution. Then*

(a) $P^* = D^*$ *(strong duality)*
(b) $(r^*, \alpha^*, p^*) \in \underset{r,\, \alpha \in \mathcal{A},\, p \in \mathcal{P}}{\text{argmin}} L(r, \alpha, p, \nu^*, \Lambda^*)$

This proposition suggests the possibility of developing an algorithm to find the optimal dual variables ν^*, Λ^*, and then via (b) recover the optimal primal variables r^*, α^*, p^*. To follow this path, first note that the Lagrangian in (14) can be rearranged as

$$L(r, \alpha, p, \nu, \Lambda) = r^2 - r\, Tr(\Lambda P) + Tr(\Lambda D_0) - \nu^T b + \mathbb{E}_h \sum_{i=1}^m \alpha_i(h)\ [\nu_i p_i(h) - Tr(\Lambda D_i)\, q(h_i, p_i(h))]. \quad (17)$$

By this expression finding the primal Lagrange optimizers in (15) is easy. By strict convexity and differentiability with respect to r, the minimizer $r(\nu, \Lambda)$ is unique and equals

$$r(\nu, \Lambda) = \min\{\, 1/2\, Tr(\Lambda P),\ r_{\max}\} \quad (18)$$

where we enforced the implicit constraint $0 \leq r \leq r_{\max}$.

Optimizing over the functions $\alpha(.), p(.)$ in (17) is also simplified because they are decoupled over channel states $h \in \mathcal{H}^m$, i.e., the integration $\mathbb{E}_h$ does not affect the optimal solution and the values $\alpha(h), p(h)$ can be found for each value h separately. Power minimizers at each $h \in \mathcal{H}^m$ are

$$p_i(\nu, \Lambda; h) = \underset{0 \leq p \leq p_{\max}}{\text{argmin}} \ \nu_i\, p - Tr(\Lambda D_i)\, q(h_i, p), \quad (19)$$

which implies a further decoupling among sensors i – see Remark 2. The optimal scheduling decision for each channel state h in (17) is obtained as

$$\alpha(\nu, \Lambda; h) = \underset{\alpha \in \Delta_m}{\text{argmin}} \ \sum_{i=1}^m \alpha_i\, \xi(h_i, \nu_i, \Lambda), \quad (20)$$

where

$$\xi(h_i, \nu_i, \Lambda) = \min_{0 \leq p \leq p_{\max}} \ \nu_i p - Tr(\Lambda D_i)\, q(h_i, p). \quad (21)$$

By the form of the probability simplex Δ_m in (2) the minimizing scheduling is deterministic. The scheduler picks with certainty the sensor with the lowest value $\xi(h_i, \nu_i, \Lambda)$ (or one of them if non-unique). This reveals the opportunistic nature of the channel-aware scheduler which, based on the current channel conditions, dynamically assigns channel access to the sensor with lowest relative value $\xi(h_i, \nu_i, \Lambda)$.

We now present an iterative algorithm to solve the dual problem. As noted earlier this is an online algorithm, hence the variables are indexed by real time steps $k \geq 0$. The iterative steps of the algorithm are as follows:

i) At time step k observe current channel conditions h_k, and given current dual variables ν_k, Λ_k, compute primal optimizers of the Lagrangian at h_k using (18)-(20) as

$$r_k = r(\nu_k, \Lambda_k) \quad (22)$$
$$p_{i,k} = p_i(\nu_k, \Lambda_k; h_k), \quad i = 1, \ldots, m, \quad (23)$$
$$\alpha_k = \alpha(\nu_k, \Lambda_k; h_k) \quad (24)$$

ii) Update the dual variables as

$$\nu_{i,k+1} = [\nu_{i,k} + \epsilon_k(\alpha_{i,k} p_{i,k} - b_i)]_+ \quad (25)$$

[1] The proofs of the results in this paper, omitted due to space limitations, can be found in [16], and are based on previous work [13].

$$\Lambda_{k+1} = [\Lambda_k + \epsilon_k (D_0 - \sum_{i=0}^{m} \alpha_{i,k}\, q(h_{i,k}, p_{i,k}) D_i - r_k P)]_+ \tag{26}$$

where $[\quad]_+$ denotes the projection on the non-negative orthant and on the positive semidefinite cone in (25) and (26) respectively, and $\epsilon_k \geq 0$ is a step size.

The intuition behind the algorithm is that dual variables are updated in (25), (26) in a random direction which in expectation is a subgradient of the dual function g. In other words, the algorithm implements an online dual subgradient method. The following proposition establishes that the algorithm converges to the optimal solution for the dual of the optimal scheduling and power allocation problem.

Proposition 2. *Consider the optimization problem (10)-(12) and its dual derived in (16). Based on a sequence $\{h_k, k \geq 0\}$ of i.i.d. random variables with distribution ϕ on $\mathcal{H}^m$, let the algorithm described in steps (i)-(ii) be employed with step sizes satisfying*

$$\sum_{k=0}^{\infty} \epsilon_k^2 < \infty, \; \sum_{k=0}^{\infty} \epsilon_k = \infty. \tag{27}$$

Then almost surely with respect to $\{h_k, k \geq 0\}$ it holds

$$\lim_{k\to\infty} (\nu_k, \Lambda_k) = (\nu^*, \Lambda^*), \quad \text{and} \quad \lim_{k\to\infty} g(\nu_k, \Lambda_k) = D^* \tag{28}$$

where ν^*, Λ^* is an optimal solution of the dual problem and D^* is the optimal value of the dual problem.

Besides optimizing over dual variables, the algorithm can be interpreted as a communication protocol of how to schedule sensors and allocate transmit power, adapting online to the observed channel conditions. The following theorem establishes the control performance guarantees provided by the communication protocol for the wireless control architecture of Section II. On the technical side, this is the only place where we enforce Assumption 2, which guarantees that the Lagrange minimizers in (19)-(20) are almost surely unique, and consequently that the optimal primal variables can be recovered from the online protocol in the limit.

Theorem 1. *Consider the wireless control architecture of Fig. 1 with plant dynamics described by (1), and a given function $V(x) = x^T P x$, $P \in \mathbb{S}^n_{++}$. Consider transmission variables $\gamma_{i,k}$ described by (3), depending on channel states $h_k \in \mathcal{H}^m$ which are i.i.d. with distribution ϕ, scheduling $\alpha_k \in \Delta_m$, and power allocation $p_k \in [0, p_{\max}]^m$. Let Assumptions 1, 2 hold. If α_k, p_k adapt to the channel sequence $h_{0:k}$ according to algorithm (22)-(26), with stepsizes ϵ_k satisfying (27), then almost surely the power consumption for each sensor i satisfies*

$$\limsup_{k\to\infty} \mathbb{E}\left[\, \alpha_{i,k} p_{i,k} \,|h_{0:k-1}\right] \leq b_i, \tag{29}$$

and the decrease rate of $V(x)$ satisfies for any $x \in \mathbb{R}^n$

$$\limsup_{k\to\infty} \mathbb{E}\left[V(x_{k+1}) \,\middle|\, x_k = x, h_{0:k-1}\right] \leq r^* V(x) + Tr(PW) \tag{30}$$

where r^* is the optimal solution of problem (10)-(12).

According to the theorem, the protocol converges almost surely to a configuration that respects the sensors' power constraints and minimizes the decrease rate of the given Lyapunov function. This however does not a priori imply system stability. If the algorithm converges to some $r^* > 1$ then the resulting communication protocol may lead to either an unstable or a stable system. This does not contradict the necessary and sufficient stability condition of (8) which states that *some* appropriate quadratic Lyapunov function exists. The online algorithm is based on a fixed function, under which stability may not be provable. If however $r^* < 1$ then indeed stability is guaranteed (cf. (8)). A necessary and sufficient condition for $r^* < 1$ is that the feasible set of problem (10)-(12) contains a point $r < 1$. We restate this observation in the following corollary.

Corollary 1. *Consider the setup of Theorem 1 and additionally suppose the optimization problem (10)-(12) contains a feasible solution with $r < 1$. Then almost surely $\mathbb{P}\left[\gamma_{i,k} | h_{0:k-1}\right]$ for $i = 1, \ldots, m$ converge to values such that system (1) is mean square stable.*

After some remarks on the structure of the communication protocol, we present numerical simulations of the online algorithm in the following section.

Remark 2. The online communication protocol implies a decentralized power allocation. In step (23), as noted in (19), the transmit power $p_{i,k}$ for sensor i, if scheduled, does not depend on the whole channel vector h_k but only on the respective channel state $h_{i,k}$, as well as on the variables $\nu_{i,k}, \Lambda_k$. Similar separability results are common in wireless communication networks [12]. From an implementation perspective, as noted in Remark 1, channel states $h_{i,k}$ can be estimated at each sensor i. The variables $\nu_{i,k}, \Lambda_k$ can be sent from the scheduler to the scheduled sensor i at each time step. As $\nu_{i,k}, \Lambda_k \to \nu_i^*, \Lambda^*$ according to Prop.2, at the limit operating point each sensor can locally store ν_i^*, Λ^* and select power according to the stored values and the current channel conditions. Note however that scheduling in (24) is centralized since, by (20), it depends on all dual variables and the channel states observed by all sensors. □

IV. Numerical Simulations

We consider the frequently used benchmark example of a batch reactor [2], [8]. The continuous time plant and controller dynamics can be found in the referred works, and involve a plant with 4 states, 2 inputs and m = 2 outputs, and a PI controller with 2 states. Under a transmission period of $0.02s$, and assuming that outputs are kept constant at the controller between transmissions (see [2]), we obtain the discrete time switched dynamics of the form (1). Then a quadratic Lyapunov function needs to be chosen. Consider a function that would guarantee stability if each sensors transmits successfully 40% of the time, e.g., satisfying

$$\sum_{i=1}^{2} 0.4\, A_i^T P A_i + 0.2\, A_0^T P A_0 = 0.98\, P - 0.001\, I. \tag{31}$$

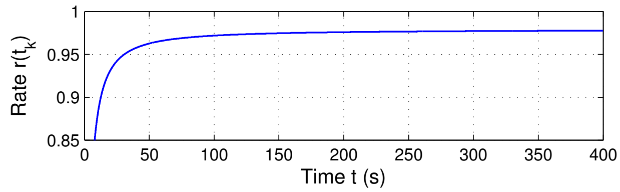

Fig. 3. Rate variable r_k during online algorithm. The variable converges to a Lyapunov decrease rate less than 1, implying mean square stability.

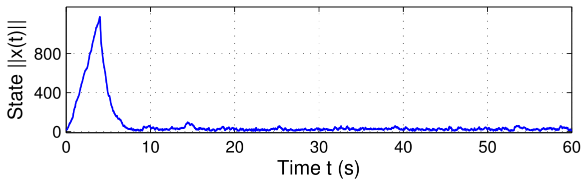

Fig. 4. Norm of system state $\|x(t)\|$ during online algorithm. The norm remains bounded, after an initial transient phase where the online algorithm has not converged to a stabilizing communication protocol.

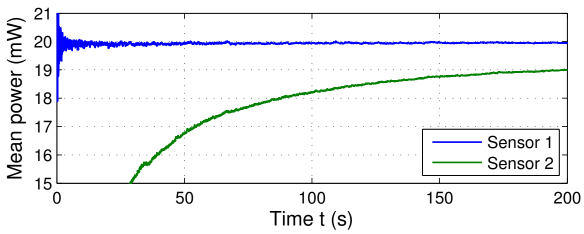

Fig. 5. Sensors' average power consumption during the online algorithm. In the limit both satisfy the power constraint $b_i = 20mW$.

The value 0.98 is selected after some trials and relates to the fact that the system has an eigenvalue very close to 1 (also documented in [2]), while the term $-0.001\,I$ guarantees the left and right hand sides are almost equal.

We model the channel gains $h_{1,k}$, $h_{2,k}$ as independent over time k and also among the two sensors, both exponentially distributed with a normalized mean 1. The maximum transmit power and the power budgets are modeled as $p_{\max} = 100mW$ and $b_i = 20mW$ respectively for both sensors. The function $q(h\,p)$ is shown in Fig. 2.

We run the online algorithm of (22)-(26) in Section III, which converges to a communication protocol where sensors 1 and 2 successfully transmit approximately 54% and 41% of the time respectively, slightly deviating from the values assumed in the Lyapunov construction (31). The transmitted packet is lost 2% of the time, and the remaining 3% accounts for times when the scheduled sensor used zero power. As shown in Fig. 3 the algorithm converges to a protocol that stabilizes the system according to (30), since the rate variable r_k tends to $r^* \approx 0.98$. Stability is also verified at the system state plot in Fig. 4. The resulting protocol meets the sensor's power constraints, as we see in Fig. 5 where we plot the mean power $1/N \sum_{k=1}^{N} \alpha_{i,k} p_{i,k}$ for each sensor i during the algorithm. Before convergence, sensor 2 does not transmit often enough or with enough power, explaining the large initial states in Fig. 4.

V. Concluding Remarks

In this paper we considered the problem of scheduling power-constrained sensors in wireless control systems. We developed a protocol where scheduling decisions and transmit power allocation are selected online based on the observed random wireless channel conditions and the objective is to obtain a configuration such that the control system is stable. The protocol is based on a given Lyapunov function, under which however the system might not be stabilizable. While a heuristic method was employed to construct a Lyapunov function in Section IV, determining Lyapunov functions suitable for the scheduling algorithm requires further examination. This relates to the controller design problem which could potentially be studied together with the communication design. Future work includes the design of schedulers adapting not only to channel but also to plant states, as in, e.g., [2], [8], as well as decentralized channel access approaches.

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